Are negative nominal interest rates expansionary?*

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Abstract

Following the crisis of 2008 several central banks engaged in a radical new policy experiment by setting negative policy rates. Using aggregate and bank level data, we document a collapse in pass-through to deposit and lending rates once the policy rate turns negative. We show that the pass-through of policy rate cuts to lending rates are weaker for banks with higher deposit shares, and that these banks have substantially lower credit growth once the central bank implements negative rates. Motivated by these empirical facts, we construct a macro-model with a banking sector that links together policy rates, deposit rates and lending rates. Once the policy rates turns negative the usual transmission mechanism of monetary policy breaks down. Moreover, because a negative interest rate on reserves reduces bank profits, the total effect on aggregate output can be contractionary.

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1 Introduction

Nominal interest rates have been declining over the past decades, resulting in record low policy rates. Several countries have interest rates close to or at zero percent, and some have gone even further. Between 2012 and 2016, a handful of central banks reduced their key policy rates below zero for the first time in history. While real interest rates have been negative on several occasions, the use of negative nominal rates prompted a new discussion on the relevance of the zero lower bound. The recent experience with negative interest rates in Japan and a number of European countries makes it clear that negative nominal interest rates should be viewed as part of the central bankers toolbox. However, the question of how negative interest rates affect the macroeconomy is largely unresolved. Our goal in this paper is to answer this question.

Understanding how negative nominal interest rates affect the economy is important in preparing for the next economic downturn. Interest rates have been declining for more than three decades, resulting in worries about secular stagnation (see e.g. Summers 2014, Eggertsson and Mehrotra 2014 and Caballero and Farhi 2017). In a recent working paper, Kiley and Roberts (2017) estimate that the zero lower bound on nominal interest rates will bind 30-40 percent of the time going forward. Whether setting a negative interest rate is expansionary is therefore of first order importance.

Why did central banks try this untested policy? In short, they argued that there is nothing special about policy rates falling below zero. When announcing a negative policy rate, the Swedish Riksbank wrote in their monetary policy report that "Cutting the repo rate below zero, at least if the cuts are in total not very large, is expected to have similar effects to repo-rate cuts when the repo rate is positive, as all channels in the transmission mechanism can be expected to be active" (The Riksbank 2015). Similarly, the Swiss National Bank declared that “the laws of economics do not change significantly when interest rates turn negative” (Jordan, 2016). Many are skeptical however. For instance, Mark Carney of the Bank of England is “… not a fan of negative interest rates” and argues that “we see the negative consequences of them through the financial system” (Carney, 2016). One such consequence might be the adverse impact on bank profitability, which has caused concern in the Euro Area in particular (Financial Times, 2016). Consistent with this view, Waller (2016) coins the policy a “tax in sheep’s clothing”, arguing that negative interest rates act as any other tax on the banking system and thus reduces credit growth\(^1\).

In this paper we investigate the impact of negative central bank rates on the macroeconomy, both from an empirical and theoretical perspective\(^2\). The first main contribution of the

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\(^1\)Other skeptics include Stiglitz (2016) and McAndrews (2015).

\(^2\)Note that we do not attempt to evaluate the impact of other monetary policy measures which occurred
paper is to use a combination of aggregate and bank level data to examine the pass-through of negative nominal central bank rates via the banking system. Using aggregate data, across six different economies, we show how negative policy rates have had limited pass-through to bank deposit rates, i.e. the rates customers face when they deposit their money in banks, and to lending rates, i.e. the rate at which customers borrow from banks. Making inferences about whether negative interest rates are expansionary based on aggregate data is challenging, however. We therefore proceed by using a unique and novel, daily bank level dataset on lending rates from Sweden, to explore the decoupling of lending rates from the policy rate. Due to the high frequency nature of the data, we are able to estimate the causal effect of policy rate cuts on bank lending rates. We can then compare the transmission mechanism of monetary policy in positive and negative territory. We document a striking decline in pass-through and a substantial increase in heterogeneity across banks for a wide range of mortgage loans once the policy rate becomes negative. We show that this increase in heterogeneity is linked to variation in the reliance on deposit financing: the higher the dependence on deposit financing, the smaller the effect on lending rates. Finally, using a difference-in-difference approach, we show that banks with high deposit shares have significantly lower credit growth once the policy rate becomes negative, consistent with similar findings for the Euro Area (Heider, Saidi, and Schepens, 2016).

Motivated by these empirical results, the second main contribution of the paper is methodological. We construct a model, building on several papers from the existing literature, which allows us to address in the most simple setting how changes to the central bank policy rate filters though the banking system to various other interest rates, and ultimately determines aggregate output. At a minimum, such a model needs to recognize the role of money as a store of value, give a role to banks in order to allow for separate lending and borrowing rates, as well as having a well defined policy rate that may differ from the rates depositors and borrowers face. We construct a simple New Keynesian DSGE model which nests the standard one-period interest rate textbook New Keynesian model (see e.g. Woodford, 2003) and other recent variations. Our framework has four main elements. First, we explicitly introduce money along with storage costs to clarify the role of money as a store of value and illustrate how this may generate a bound on deposit rates. Second, we incorporate a banking sector and nominal frictions along the lines of Benigno, Eggertsson, and Romei (2014), which delivers well defined deposit and lending rates. Third, we incorporate demand for central bank reserves as in Curdia and Woodford (2011) in order to obtain a policy rate which can potentially differ from the commercial bank deposit rate. Fourth, we allow for the possibility simultaneously with negative interest rates. That is, we focus exclusively on the effect of negative interest rates, and do not attempt to address the effectiveness of asset purchase programs or programs intended to provide banks with cheap financing (such as the TLTRO program initiated by the ECB).
that the cost of bank intermediation depends on banks’ net worth as in Gertler and Kiyotaki (2010). The central bank sets the interest rate on reserves, and can choose to implement a negative policy rate as banks are willing to pay for the transaction services provided by reserves. However, due to the possibility of using money to store value, the deposit rate faced by commercial bank depositors is bounded at some level (possibly negative), in line with our empirical findings. The reason is simple: the bank’s customers will choose to store their wealth in terms of paper currency if charged too much by the bank. We stipulate explicit conditions on the storage cost of money that guarantees a well-defined lower bound.

Away from the lower bound on the deposit rate, the central bank can stimulate the economy by lowering the policy rate. This reduces both the deposit rate and the rate at which households can borrow, thereby increasing demand. We show however, that once the deposit rate reaches its effective lower bound, reducing the policy rate further is no longer expansionary. As the central bank looses its ability to control the deposit rate, it cannot stimulate the demand of savers via the traditional intertemporal substitution channel. Furthermore, as banks’ funding costs (via deposits) are no longer responsive to the policy rate, the bank lending channel of monetary policy breaks down. There is no stimulative effect via lower borrowing rates. Hence, as long as the deposit rate is bounded, a negative central bank rate fails to bring the economy out of a recession. We further show that if bank profits affect banks’ intermediation costs, due to for instance informational asymmetries between the bank and its creditors as in Gertler and Kiyotaki (2010), negative interest rates can be contractionary through a reduction in banks’ net worth.

**Literature Review** Jackson (2015) and Bech and Malkhozov (2016) document the limited pass-through of negative policy rates to aggregate bank rates, but do not evaluate the effects on the macroeconomy. Heider, Saidi, and Schepens (2016) and Basten and Mariathasan (2018) document that negative policy rates has not lead to negative deposit rates in the Euro Area and Switzerland, respectively. While Basten and Mariathasan (2018) find that Swiss banks primarily reduce reserves in response to negative rates, Heider, Saidi, and Schepens (2016) find that banks with higher deposit shares have lower lending growth in the post-zero environment. While Heider, Saidi, and Schepens (2016) argue that this is a result of the lower bound on deposit rates, no attempt is made to formalize the mechanisms at play. We

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3 There are other reasons why there might be a lower bound on the deposit rate, which we do not explore in this paper. Rather, we choose to introduce a lower bound as a consequence of the combination of money as a store of value and storage costs, motivated by both the existing literature and empirical evidence suggesting that households would withdraw their cash had they faced a negative interest rate, see Figure 2 in Appendix A. The existence of a zero lower bound due to potential cash withdrawals is also consistent with the observation we make that the bound appears strongest for household depositors. The reason why there exists a lower bound is an interesting and important question, however it is not important for our results.
contribute to the empirical literature on the pass-through of negative rates by i) providing a comprehensive analysis of the pass-through of negative policy rates to aggregate lending and deposit rates across all relevant countries, and more importantly ii) exploit a unique dataset on lending rates to provide novel micro level evidence on the decoupling of lending rates from the policy rate. Furthermore, we show how the lack of pass-through to lending rates can be explained by cross-sectional variation in the reliance on deposit financing.

Given the radical nature of the policy experiment pursued by several central banks, the theoretical literature is perhaps surprisingly silent on the expected effect of this policy. Two important exceptions are Brunnermeier and Koby (2016) and Rognlie (2015). Following the introduction of negative rates, there were concerns that bank profits would be harmed and the policy as such could be contractionary. Brunnermeier and Koby (2016) models related concerns, by defining the reversal rate as the interest rate at which further policy rate reductions increase commercial bank lending rates due to the negative effect of reduced net interest margins. This reversal rate, however, can in principle be either positive or negative, so it is unrelated to the observed lower bound on deposit rates, which is central to both the empirical and theoretical analysis of our paper. Furthermore, Brunnermeier and Koby (2016) do not analyze the general equilibrium impact of interest rate reductions on output and inflation. Rognlie (2015) allows for a negative interest rate due to money storage costs. However, in his model households face only one interest rate, and the central bank can control this interest rate directly. Thus, the model does not allow for a separate bound on deposit rates. Hence, neither of these papers capture the key mechanism in our paper, which is driven by an empirical observation which was by no means obvious ex ante: as the lower bound on the commercial bank deposit rate becomes binding, the connection between the central bank’s policy rate (which can be negative) and the rest of the interest rates in the economy breaks down. Our main theoretical contribution is therefore building a model that can deliver this mechanism and then, in the context of our model, analyzing the effect of negative policy rates on aggregate output and inflation.

There also exists an older literature, dating at least back to the work of Silvio Gesell more than a hundred years ago, which contemplates more radical monetary policy regime changes than we do here (Gesell, 1916). This literature has been rapidly growing in recent years. In our model, the storage cost of money, and hence the lower bound, is treated as fixed. However, policy reforms could potentially alter the lower bound or even remove it.

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4 There is however a large literature on the effects of the zero lower bound. See for example Krugman (1998) and Eggertsson and Woodford (2006) for two early contributions.

5 Our paper is also related to an empirical literature on the connection between interest rate levels and bank profits (Borio and Gambacorta 2017, Kerbl and Sigmund 2017), as well as a theoretical literature linking credit supply to banks net worth (Holmstrom and Tirole 1997, Gertler and Kiyotaki 2010).
completely. An example of such policies is a direct tax on paper currency, as proposed first by Gesell and discussed in detail by Goodfriend (2000) and Buiter and Panigirtzoglou (2003). Another possibility is abolishing paper currency altogether. This policy is discussed in, among others, Agarwal and Kimball (2015), Rogoff (2017a) and Rogoff (2017c), who also suggest more elaborate policy regimes to circumvent the ZLB. The results presented here should not be considered as rebuffing any of these ideas. Rather, we are simply pointing out that under the current institutional framework, empirical evidence and a stylized variation of the standard New Keynesian model do not seem to support the idea that a negative interest rate policy is an effective tool to stimulate aggregate demand. This should, in fact, be read as a motivation to study further more radical proposals such as those presented by Gesell over a century ago and more recently in the work of authors such as Goodfriend (2000), Buiter and Panigirtzoglou (2003), Rogoff (2017c) and Agarwal and Kimball (2015).

2 Negative Interest Rates In Practice

2.1 Aggregate evidence

In this section, we use aggregate data retrieved from central banks and statistical agencies for each of the six economies we discuss, to shed light on the evolution of aggregate deposit and lending rates following the introduction of negative policy rates.

Deposit rates In Figure 1 we plot deposit rates for six economic areas in which the policy rate is negative. Starting in the upper left corner, the Swedish central bank lowered its key policy rate below zero in February 2015. Deposit rates, which in Sweden are usually below the policy rate, did not follow the central bank rate into negative territory. Instead, deposit rates for both households and firms remain stuck at, or just above, zero. A similar picture emerges for Denmark, as illustrated in the upper right corner. The Danish central bank crossed the zero lower bound twice, first in July 2012 and then in September 2014. As was the case for Sweden, the negative policy rate has not been transmitted to household deposit rates. For corporations however, the average deposit rate is slightly below zero.

Consider next the Swiss and Japanese case in the middle row of Figure 1. Switzerland implemented a negative policy rate in December 2014, while the central bank in Japan lowered its key policy rate below zero in early 2016. The deposit rates in both countries were already very low, and did not follow the policy rate into negative territory. As a result, the impact on deposit rates was limited.
Figure 1: Aggregate deposit rates for Sweden, Denmark, Switzerland, Japan, the Euro Area and Germany. The policy rates are defined as the Repo Rate (Sweden), the Certificates of Deposit Rate (Denmark), SARON (Switzerland), the Uncollaterized Overnight Call Rate (Japan) and the Deposit Rate (Euro Area and Germany). The red vertical lines mark the month in which policy rates became negative. Source: The Riksbank, Statistics Sweden, the Danish National Bank (DNB), the Swiss National Bank (SNB), Bank of Japan, and the European Central Bank (ECB).

Finally, interest rates for the Euro Area are depicted in the bottom row of Figure 1. The ECB reduced its key policy rate below zero in June 2014. As seen from the left panel, aggregate deposit rates are high in the Euro Area and therefore have more room to fall before reaching the zero lower bound. Looking at Germany only, in the bottom right of the figure, we see that deposit rates are somewhat lower. Although the corporate deposit rate in Germany appears to have dipped slightly below zero on some occasions, the Euro Area interest rate data again supports the notion that deposit rates appear bounded at some level close to zero.

Note that because banks have generally been reluctant to pass negative interest rates on to their clients, especially to households, we do not know to what extent negative deposit
rates would lead to an outflow of deposits. However, in a recent survey by the ING, 76% of the respondents said they would withdraw their money if the interest rate on their saving account turned negative (ING, 2015) see Figure 2.

![Figure 2: Fraction of households who would withdraw money from their savings account if they were levied a negative interest rate. Solid line represent unweighted average of 76.4 %. Source: ING (2015)](image)

**Lending rates** Although most deposit rates appear bounded by zero, one might still expect negative policy rates to lower *lending* rates. As lending rates are usually above the central bank policy rate, they are all well above zero. Here we show that the pass-through of the policy rate to lending rates appears weakened when the policy rate becomes negative, an empirical finding our model will replicate. In Figure 3 we plot bank lending rates for the six economic areas considered above. While lending rates usually follow the policy rate closely, there appears to be a disconnect once the policy rate breaks the zero lower bound, a feature which will become starker once we consider disaggregated bank data. Looking at the aggregate data in Figure 3, lending rates in Sweden, Denmark and Switzerland seem less sensitive to the respective policy rates once they become negative. There appears to be some reduction in Japanese lending rates at the time the policy rate went negative, but because there are no further interest rate reductions in negative territory the Japanese case is less informative\(^6\). Again, the Euro Area is somewhat of an outlier, as lending rates have decreased\(^7\). This is not surprising however, in light of the higher-than-zero deposit rates

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\(^6\)The initial reduction in Japanese lending rates could be caused by the positive part of the policy rate cut, i.e. going from a positive policy rate to a zero policy rate.

\(^7\)When considering the Euro Area it is worth noting that the negative interest rate policy was implemented together with a host of other credit easing measures, some of which implied direct lending from the ECB to commercial banks at a (potentially) negative interest rate. That policy is better characterized as a credit subsidy rather than charging interest on reserves, which the commercial banks hold in positive amounts at the central bank.
we documented in the previous fact. Again, for the case of Germany, in which the zero lower bound on the deposit rate is binding for household deposits, lending rates appear less responsive. For the countries in which the deposit rate seems to have reached its lower bound, the limited reduction in lending rates means that the net interest rate margin is largely unaffected.

Figure 3: Aggregate lending rates for Sweden, Denmark, Switzerland, Japan, the Euro Area and Germany. The policy rates are defined as the Repo Rate (Sweden), the Certificates of Deposit Rate (Denmark), SARON (Switzerland), the Uncollateralized Overnight Call Rate (Japan) and the Deposit Rate (Euro Area and Germany). The red vertical lines mark the month in which policy rates became negative. Source: The Riksbank, Statistics Sweden, the DNB, the SNB, Bank of Japan, and the ECB.

2.2 Bank-level evidence on the pass-through of negative interest rates

To better understand the pass-through of negative policy rates to bank-level rates, we proceed by using two bank level datasets for Swedish banks. First, we use daily bank level data on
a rich set of mortgages for the largest Swedish banks, which was provided by the price comparison site compricer.se. We exploit the high frequency of the data to evaluate the causal effect of reductions in the policy rate, and compare the monetary policy transmission to lending rates across positive and negative territory. Second, we complement our analysis by using bank level data on monthly lending volumes from Statistics Sweden.

**Limited pass-through to lending rates** In Figure 4 we plot daily bank level mortgage rates for the largest Swedish banks. We show in Figure 17 in Appendix A that the bank level data can be aggregated up to match the official aggregate mortgage rate data. The vertical lines in Figure 4 capture days on which the key policy rate (the repo rate) was lowered. The level of the repo rate is reported on the x-axis. The first two lines capture repo rate reductions in positive territory. On both of these occasions, there is an immediate and homogeneous decline in bank lending rates. The next line marks the day the repo rate turned negative for the first time, and the three proceeding lines capture repo rate reductions in negative territory. The response in bank lending rates to these interest rate cuts are strikingly different. While there is some initial reduction in lending rates, most of the rates increase again shortly thereafter. As a result, the total impact on lending rates is limited.

![Swedish Bank Lending Rates](image)

**Figure 4:** Bank-level lending rates in Sweden. Interest rate on mortgages with five-year fixed interest period. The red vertical lines mark days in which the repo rate was lowered. The label on the x-axis shows the value of the repo rate. Small x’s denote the change in the deposit rate relative to the change in the policy rate (%), measured on the right y-axis.

We also include in the graph the correlation between the repo rate and the aggregate

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8We have mortgage rates for different periods of fixed interest rates, ranging from one to five years, in addition to floating rates. The fixed interest rate periods for which all financial institutions provide interest rates are 1 year, 2 years, 3 years and 5 years.
deposit rate, as illustrated by the black x’es measured on the right y-axis. For the first four repo rate reductions in our sample, the change in the aggregate deposit rate was roughly 40 percent of the change in the repo rate. For the last two repo rate cuts however, the change in the aggregate deposit rate was much smaller. This highlights a subtle, but important, point: the pass-through to lending rates is smaller once the deposit rate is no longer responding to further policy rate cuts. This may be exactly when the policy rate goes negative, or as in the Swedish case, shortly thereafter. For the two last repo rate cuts in our sample, there is virtually no pass-through to either the aggregate deposit rate or to bank level mortgage rates.

In Figure 5 we depict box plots of bank level correlations between the lending rates and the policy rate. The black box to the left corresponds to the empirical distribution of correlations prior to the Riksbank going negative. The gray box in the middle corresponds to the empirical distribution for the full period of negative rates. Finally, the blue box to the right corresponds to the empirical distribution of correlations after the deposit rate becomes unresponsive to changes in the repo rate (i.e. the last two policy rate cuts). Consistent with the previous figure, there is a substantial drop in the correlation between bank lending rates and the repo rate once the repo rate becomes negative. This is especially clear for the last two repo cuts, in which deposit rates were essentially bounded. In this case, the average correlation between bank lending rates and the repo rate actually falls slightly below zero.

![Figure 5: The distribution of bank-level correlations between changes in lending rates and the repo-rate when the repo rate is positive (“positive rates”), the repo rate is negative (“negative rates”) and the repo rate is negative and the deposit rate is non-responsive (“Negative rates (bound)”). 5-year fixed interest rate period.](image)

The stark reduction in pass-through holds across a wide range of loan types. In Figure 6 we plot bank-level lending rates across three different contracts, a floating rate mortgage
(3m), a mortgage with a 1 year fixed-rate period (1y) and a mortgage with a 3 year fixed-rate period (3y). In all three cases, we see that the interest rate cuts in negative territory have very limited pass-through to bank lending rates.

Figure 6: Bank level lending rates with a floating interest rate (3m) (left panel) and a fixed interest rate period of 1y (mid panel) and a fixed interest rate period of 3y (right panel). The red solid line capture days with repo rate reductions.

In Table 1 we formally test the pass-through of changes in the repo rate to changes in bank lending rates. We make two comparisons, one where we compare the correlation pre- and post negative rates and one where we compare the correlation prior to negative rates with the correlation for the last two repo cuts, where the pass-through to the aggregate deposit rate was below 10 percent. As seen in the table, the correlation is significantly lower after the introduction of negative interest rates (Panel A) and after the deposit rate reaches its lower bound (Panel B). The reduction is pass-through is statistically significant across all fixed interest-rate periods. Moreover, the drop in correlation is especially large when comparing the pass-through prior to negative rates with the pass-through during the two final policy rate cuts.

<table>
<thead>
<tr>
<th>Fixed interest rate period:</th>
<th>Floating rate</th>
<th>1y</th>
<th>3y</th>
<th>5y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean correlation, before</td>
<td>0.92</td>
<td>0.91</td>
<td>0.82</td>
<td>0.76</td>
</tr>
<tr>
<td>Mean correlation, after</td>
<td>0.57</td>
<td>0.58</td>
<td>0.63</td>
<td>0.53</td>
</tr>
<tr>
<td>Mean correlation, after bound</td>
<td>0.37</td>
<td>0.27</td>
<td>0.15</td>
<td>−0.01</td>
</tr>
</tbody>
</table>

Panel A. Difference: After - before  

|                       | −0.35***     | −0.33*** | −0.19*** | −0.22*** |
|                       | 0.00         | 0.00     | 0.00      | 0.00     |

Panel B. Difference: After bound - before  

|                       | −0.54**      | −0.63*** | −0.67*** | −0.77*** |
|                       | 0.05         | 0.00     | 0.00      | 0.00     |

Table 1: Correlation between changes in the repo rate and banks lending rates at different maturities. Data collapsed to monthly mean. “After” refer to the time period after the repo rate was first reduced below zero. “After (bound)” refer to the two most recent repo rate cut, where the deposit rate is essentially bounded at zero.
Increased dispersion in bank lending rates  Negative policy rates also leads to a substantial increase in the dispersion of lending rates across banks. In the left panel of Figure 7, we plot the minimum and maximum bank lending rate along side the repo rate (the dashed black line). The increase in dispersion after the repo rate turned negative is clearly visible. We also note that the minimum bank lending rate has stayed constant since the first quarter of 2015, despite three policy rate reductions in negative territory. In the right panel of Figure 7, we illustrate the increase in dispersion explicitly by plotting the standard deviation of lending rates over time. We first note that the dispersion in bank rates appears to spike around the time when changes to the repo rate are announced. Second, and more importantly for our purpose, there is a sustained increase in dispersion after the zero lower bound is breached.

What is causing the increase in dispersion in banks’ responses to policy rate cuts? One theory is that differences in the reliance on deposit financing means that banks are being differentially affected by negative interest rates. Given that there are frictions in raising different forms of financing - and some sources of financing are more responsive to monetary policy changes than others - cross-sectional variation in balance-sheet components can induce variation in how monetary policy affects banks (Kashyap and Stein, 2000). This is especially relevant in our setting. Negative interest rates have had a more limited pass-through to deposit financing relative to other sources of financing. To investigate whether banks’ funding structures affect their willingness to lower lending rates, we plot the bank level correlation between lending rates and the repo rate after the repo rate turned negative, as a function of banks’ deposit shares. The result is depicted in Figure 8. As is clear from the figure, there is a negative relationship between the deposit share and the correlation with the repo rate. That is, banks with higher deposit shares are less responsive to policy rate cuts in negative
Weighting observations by market shares, this relationship is statistically significant at the one percent level. The regression line indicates that a 10 percentage points increase in the deposit share is associated with a reduction in the correlation of approximately 0.17 correlation points.

Figure 8: Correlation between lending rate and repo rate after the repo rate turned negative and the deposit rate reached its lower bound, as a function of the banks’ deposit share. Size of circles indicate market share. Gray square indicates Alandsbanken, for which we do not have the market share. Regression coefficient (standard.error) also reported. *** indicates $p < 0.01$. Swedish banks. Interest rate on 5 year mortgages.

So far we have investigated what happens to bank interest rates, or prices. An alternative approach is to instead consider quantities, or bank lending volumes. This is the approach adopted in Heider, Saidi, and Schepens (2016) for Euro Area banks. Consistent with our proposed explanation, they find that banks with higher deposit shares had lower lending growth after the policy rate turned negative. Here we show that this result also holds for Swedish banks. Following Heider, Saidi, and Schepens (2016) we use the difference in difference framework specified in equation (1). The dependent variable is the (approximate) percentage 3-month growth in bank level lending. $I_{i}^{post}$ is an indicator variable equal to one after the policy rate became negative, while Deposit share$_i$ is the deposit share of bank $i$ in year 2013. As an alternative specification, we replace Deposit share$_i$ with an indicator $1_{High \ deposit\ share}$ for whether bank $i$ has a deposit share above the median in 2013. We include bank fixed effects $\delta_i$ to absorb time-invariant bank characteristics, and month-year fixed effects $\delta_t$ to absorb shocks common to all banks. Standard errors are clustered at the bank level. We restrict our sample to start in 2014, following Heider, Saidi, and Schepens (2016)
in choosing a relatively short time period around the event date. The coefficient of interest is the interaction coefficient \( \beta \). If banks with high deposit shares have lower credit growth than banks with low deposit shares after the policy rate breaches the zero lower bound, we expect to find \( \hat{\beta} < 0^{10} \).

\[
\Delta \log(\text{Lending}_{i,t}) = \alpha + \beta (I_{t}^{\text{post}} \times \text{Deposit share}_i) + \delta_i + \delta_t + \epsilon_{it}
\] (1)

The regression results are reported in Table 2. Focusing on column (1) first, the interaction coefficient is negative as expected, and significant at the five percent level. An increase in the deposit share is associated with a reduction in credit growth at the bank level. Stated differently, credit growth in the post-zero environment is significantly lower for banks which rely heavily on deposit financing. The effect is economically significant - a one standard deviation increase in the deposit share decreases lending growth by approximately 0.18 standard deviations.

In column (2) we look at the average growth in credit for high deposit share banks relative to low deposit share banks in the post-zero environment. While we lose some precision by using only an indicator variable, the coefficient is still negative and statistically significant at the ten percent level. On average, banks with a high deposit share had four percentage points lower growth in credit compared to banks with a low deposit share. We thus conclude that, due to the lower bound on the deposit rate, banks which rely heavily on deposit financing are less responsive to policy rate cuts in negative territory. The cross-sectional evidence presented here, is consistent with the survey evidence in Figure 9 where the vast majority of European banks report that they have not increased lending volumes in response to negative policy rates.

\[^{10}\text{In a previous version of the paper, we used 1-month growth in bank-level lending instead of 3-month growth. In both cases our coefficient estimates are negative and statistically significant. However, using 3-month growth rates is more consistent with the analysis in Heider, Saidi, and Schepens (2016) based on quarterly data, and increases our precision slightly.}\]
Dependent variable: $\Delta \log (Lending)_{i,t}$

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
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<tbody>
<tr>
<td>$I_{t}^{post} \times \text{Deposit share}_{i}$</td>
<td>$-0.08^{**}$</td>
<td>$-0.04^*$</td>
</tr>
<tr>
<td></td>
<td>(-2.09)</td>
<td>(-1.85)</td>
</tr>
<tr>
<td>$I_{t}^{post} \times 1_{\text{High deposit share}_{i}}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Clusters 40 40
Bank FE Yes Yes
Month-Year FE Yes Yes
Observations 1113 1113

Table 2: Regression results from estimating equation (1). Dependent variable: $\Delta \log (Lending)_{i,t} \equiv \log (Lending)_{i,t} - \log (Lending)_{i,t-3}$. Monthly bank level data from Sweden.

Overall funding costs  It is possible that the effective deposit rate has fallen more than the nominal deposit rate. This would be the case if banks have responded to negative interest rates by increasing fees. We think this is of limited quantitative importance for two reasons. First, if the lower bound is due to the existence of money as a store of value, there is no reason to believe that the effective deposit rate should not be subject to the same lower bound. Second, as we show in Figure 10 aggregate commission income for Swedish banks did not increase following the introduction of negative policy rates.
The empirical analysis so far has centered around the link between deposit rates and lending rates. However, banks also use other financing sources. In the left panel of Figure 11 we show the composition of liabilities for Swedish banks. Deposits account for almost half of banks total financing, while covered bonds accounts for almost 1/4. Remaining liabilities are accounted for by interbank loans, certificates and unsecured debt. In the right panel of Figure 11 we plot various other interest rates. Interbank rates, represented by STIBOR 3M, have largely followed the repo rate into negative territory. However, as net interbank lending is small in aggregate this has had limited impact on banks overall funding costs. The pass-through of negative rates to long-term covered bonds appears substantially weakened. Interest rates on short-term covered bonds, however are slightly negative.

What does this imply for overall funding costs? In Figure 12 we plot an estimate of the average funding costs for Swedish banks over time. The estimate is the weighted average of
the interest rates on the various liabilities of Swedish banks. For those liabilities where we do not observe the interest rate (unsecured debt and certificates), we assume that the pass-through equals the pass-through to short-term covered bonds. Because the pass-through to short-term covered bonds has been fairly strong we view this as an conservative assumption\textsuperscript{11}. As is clear from the figure, the sensitivity of total average financing costs to the policy rate appears substantially weakened once the policy rate goes negative. This is the result of the sharp reduction in the pass-through to the two largest financing sources for Swedish banks, deposits and covered bonds.

To illustrate the reduction in pass-through, we construct a counterfactual funding cost where the markup relative to the repo rate equals the average markup in the pre-zero period. Our estimate suggests that if there had not been a decline in pass-through, the average funding cost for Swedish banks would be slightly negative, compared to approximately 0.25 percent today.

![Figure 12: Estimated average funding costs. The estimated average funding cost is computed by taking the weighted average of the assumed interest rate of the different funding sources of the bank. Certificates are assumed to have the same interest rate as 2Y covered bonds, while unsecured debt are assumed to have the same interest rate as 2Y covered bonds plus a 2 percent constant risk-premium. The “counterfactual” series correspond to the case when the spread between the repo rate and the estimated funding cost remain fixed at pre-negative levels. Weights based on the liability structure of large Swedish banks, see Figure 11 in Appendix A.](image)

\textsuperscript{11}An even more conservative assumption would be assuming full pass-through. In Appendix A, we construct an alternative estimate based on this assumption. Although the pass-through becomes slightly larger in this case, the decline in pass-through is still striking.
Figure 13, we plot new issuances of covered bonds - the most important non-deposit financing source. There does not appear to have been an increase in covered bond issuances after the repo rate went negative. Perhaps surprisingly, the right panel of Figure 12 depicts an increase in deposit financing in the post-zero environment.

Why do Swedish banks remain reliant on deposit financing, despite their relatively high cost? There are at least three possible explanations: i) maintaining a base of depositors creates some synergies which other financing sources do not, ii) the room for new issuances of covered bonds may be limited by the availability of bank assets to use for collateral, iii) Basel III regulation makes deposit financing more attractive in terms of satisfying new requirements. We do not take a stand on why banks have increased their deposit financing, but note that the empirical evidence suggests that deposit rates are the most important component of, not only average, but also marginal funding costs. In our model we therefore make the simplifying assumption that deposits is the only financing source.

3 Negative interest rates in theory

Motivated by the empirical facts outlined in the previous section, we now develop a formal framework for understanding the impact of negative central bank policy rates.

3.1 Model

To theoretically analyze how effective negative interest rates are in terms of stabilizing inflation and output, we set up a New Keynesian (NK) model. Several parts of the model are standard in the literature, and a detailed discussion and derivation of the full set of equilibrium conditions is relegated to Appendix C. In this section, we provide an overview
of the main elements and discuss how the log-linearized version of our model can be seen as a generalization of the standard New Keynesian model with an endogenous natural rate of interest. A key new element is explicit account of interest on reserves, a separate bound on the deposit rate that affects funding costs, a relationship between bank net worth and funding costs, and an explicit account of the ZLB due to the existence of money as store of value.

The model consists of four main parts. First, there is a household sector in which households differ in their time preferences. In equilibrium, patient households are savers, and impatient households are borrowers. In the main text we focus on the money demand part of the household problem, as this gives rise to the lower bound on the deposit rate. The second building block of the model is the firm sector, which builds heavily on Benigno, Eggertsson, and Romei (2014). Firms produce using labor and face price rigidities in the form of Calvo pricing. Because the firm problem is standard we do not discuss it further in the main text. The third part of our model is the bank sector. This is a crucial part of our model, and contains mostly novel elements. Accordingly we outline the bank problem in some detail below. Finally, we introduce central bank reserves and outline the necessary assumptions about policy to close the model.

Households demand for money and the lower bound

Household $j \in \{s, b\}$ consumes, holds money, saves and supplies labor. Households of type $s$ are savers, while households of type $b$ are borrowers. We let $\Omega \left( \frac{M^j_t}{P_t} \right)$ denote the utility from holding real money balances. $U \left( C^j_t \right)$ denotes the utility of consumption and $S \left( M^j_t \right)$ denotes the storage cost of holding money. Furthermore, let $i^j_t$ be the interest rate that an agent of type $j$ faces. In our set-up, optimal money holdings have to satisfy

$$\frac{\Omega' \left( \frac{M^j_t}{P_t} \right)}{U' \left( C^j_t \right)} = \frac{i^j_t + S' \left( M^j_t \right)}{1 + i^j_t}$$  \hspace{1cm} (2)

Households demand for money depends on the marginal utility from holding money, as well as the marginal cost. The latter depends not only on the opportunity cost of lost interest income, but also on the marginal storage cost $S' \left( M^j_t \right)$. The lower bound on the deposit rate $\bar{i}^j$ is typically defined as the lowest value of $i^j_t$ satisfying equation (2). The lower bound therefore depends crucially on the marginal storage cost. We assume proportional storage

\textsuperscript{12}We assume a satiation point for money. That is, at some level $\bar{m}^j$ households become satiated in real money balances, and so $\Omega' (\bar{m}^j) = 0$.\textsuperscript{12}
cost $S(M^*_t) = \gamma M^*_t$, which implies a lower bound on the deposit rate of $i^*_s = -\gamma$ (as $\Omega'(M^*_t/P_t) = 0$ when households are satiated in real money balances). This allows for the possibility that the lower bound is negative ($\gamma > 0$), and also nests the case of a lower bound at exactly zero ($\gamma = 0$).

The role of banks

Our banking sector is made up of identical, perfectly competitive banks. Bank assets consist of one-period real loans $l_t$. In addition to loans, banks hold real reserves $R_t \geq 0$ and real money balances $m_t = M_t/P_t \geq 0$, both issued by the central bank\(^{13}\). Bank liabilities consist of real deposits $d_t$. Reserves are remunerated at the interest rate $i_t^r$, which is set by the central bank. Loans earn a return $i_t^b$. The cost of funds, i.e. the deposit rate, is denoted $i_t^s$. Banks take all of these interest rates as given.

Financial intermediation takes up real resources. Therefore, in equilibrium, there is a spread between the deposit rate $i_t^s$ and the lending rate $i_t^b$. We assume that banks’ intermediation costs are given by a function $\Gamma(l_t, R_t, m_t, z_t)$, where $z_t = Z_t/P_t$ is real bank profits. In order to allow for the intermediation cost to be time-varying for a given set of bank characteristics, we include a stochastic cost-shifter $\ell_t$. This cost-shifter may capture time-variation in borrowers default probabilities, changes in the borrowing capacity, bank regulation etc. (Benigno, Eggertsson, and Romei, 2014).

We assume that the intermediation cost is increasing and convex in the amount of real loans provided. That is, $\Gamma_l > 0$ and $\Gamma_{ll} \geq 0$. Central bank currency plays a key role in reducing intermediation costs\(^{14}\). The marginal cost reductions from holding reserves and money are captured by $\Gamma_R \leq 0$ and $\Gamma_m \leq 0$ respectively. We assume that the bank becomes satiated in reserves for some level $\bar{R}$. That is, $\Gamma_R = 0$ for $R \geq \bar{R}$. Similarly, banks become satiated in money at some level $\bar{m}$, so that $\Gamma_m = 0$ for $m \geq \bar{m}$. Banks can thus reduce their intermediation costs by holding reserves and/or cash, but the opportunity for cost reduction can be exhausted. Finally, we allow for the possibility that higher profits may reduce the marginal cost of lending. That is, we assume $\Gamma_{lz} \leq 0$. We discuss this assumption below.

Following Curdia and Woodford (2011) and Benigno, Eggertsson, and Romei (2014) we assume that any real profits from the bank’s asset holdings are distributed to their owners in period $t$ and that the bank holds exactly enough assets at the end of the period to pay off

\(^{13}\)Because we treat the bank problem as static - as outlined below - we can express the maximization problem in real terms.

\(^{14}\)For example, we can think about this as capturing in a reduced form way the liquidity risk that banks face. When banks provide loans, they take on costly liquidity risk because the deposits created when the loans are made have a stochastic point of withdrawal. More reserves helps reduce this expected cost.
the depositors in period $t+1$. Furthermore, we assume that the bank has the same storage costs of money as the household sector. Under these assumptions, real bank profits can be implicitly expressed as:

$$z_t = \frac{i_t^b - i_t^s}{1 + i_t^s} l_t - \frac{i_t^s - i_t^r}{1 + i_t^s} R_t - \frac{i_t^s + \gamma}{1 + i_t^s} m_t - \Gamma \left( \frac{l_t}{l_t^*}, R_t, m_t, z_t \right)$$  \hspace{2cm} (3)

Any interior $l_t$, $R_t$ and $m_t$ have to satisfy the respective first-order conditions from the bank’s optimization problem

$$l_t : \frac{i_t^b - i_t^s}{1 + i_t^s} = \frac{1}{\Gamma_l} \Gamma_l \left( \frac{l_t}{l_t^*}, R_t, m_t, z_t \right)$$  \hspace{2cm} (4)

$$R_t : -\Gamma_R \left( \frac{l_t}{l_t^*}, R_t, m_t, z_t \right) = \frac{i_t^s - i_t^r}{1 + i_t^s}$$  \hspace{2cm} (5)

$$m_t : -\Gamma_m \left( \frac{l_t}{l_t^*}, R_t, m_t, z_t \right) = \frac{i_t^s + \gamma}{1 + i_t^s}$$  \hspace{2cm} (6)

The first-order condition for real loans says that banks trade off the marginal income from lending with the marginal increase in intermediation costs. The next two first-order conditions describe banks demand for reserves and cash. We assume that reserves and money are not perfect substitutes, and so minimizing the intermediation cost implies holding both reserves and money.

The first-order condition for loans pins down the equilibrium credit spread $\omega_t$, defined as

$$\omega_t \equiv \frac{1 + i_t^b}{1 + i_t^s} - 1 = \frac{i_t^b - i_t^s}{1 + i_t^s}$$  \hspace{2cm} (7)

Specifically, if borrowers are of measure $\chi$, the equilibrium credit spread is

$$\omega_t = \frac{1}{\chi b_t} \Gamma_l \left( \frac{b_t}{b_t^*}, R_t, m_t, z_t \right)$$  \hspace{2cm} (8)

where we have used the market clearing condition $l_t = \chi b_t^{b_t}$ to express the spread as a function of the borrowers real debt holdings $b_t^{b_t}$. That is, the difference between the borrowing rate and the deposit rate is an increasing function of the aggregate relative debt level, and a decreasing function of bank profits.

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15The latter is equivalent to assuming that $(1 + i_t^b) l_t + (1 + i_t^r) R_t + m_t - S (m_t) = (1 + i_t^s) d_t$.

16Assuming that $\Gamma \left( \frac{l_t}{l_t^*}, R_t, m_t, z_t \right)$ is such that there exists a unique $z$ solving equation (3).

17We also assume that $l_t = \chi b_t$. 
Why do bank profits affect intermediation costs? We have allowed for the possibility that the marginal cost of extending loans decreases in bank profits. That is, $\Gamma_{iz} \leq 0$. This assumption captures, in a reduced form manner, the established link between banks’ net worth and their operational costs - assuming there is a one-to-one mapping between net worth and profits. We do not make an attempt to microfound this assumption, which is explicitly done in among others Holmstrom and Tirole (1997) and Gertler and Kiyotaki (2010), as well as documented empirically in for instance Jiménez, Ongena, Peydró, and Saurina (2012).

If $\Gamma_{iz} = 0$, there is no feedback effect from bank profits to credit supply. Note that this does not change the main result that negative interest rates are not expansionary, but it implies that there is not contractionary effect of negative interest rates. Because there has been widespread concern that negative interest rates would hurt bank profitability and thereby dampen credit supply (see for instance Waller (2016) and Financial Times (2016)), we also allow for the possibility that $\Gamma_{iz} < 0$.

Policy

To close the model, we need to be explicit about policy. The central bank in our model sets the interest rate on reserves $i_r^*$ and chooses the total supply of central bank currency in order to ensure that $i_s^* = i_r^*$ whenever possible. From the first order condition for reserves (5), we see that $i_s^* = i_r^*$ implies that $\Gamma_R = 0$. Hence, as long as banks are satiated in reserves, the central bank implicitly controls $i_s^*$ via $i_r^*$. A key point however, is that $\Gamma_R = 0$ is not always feasible due to the lower bound on the deposit rate. If the deposit rate is bounded at $i_s = -\gamma$, and the central bank lowers $i_r^*$ below $-\gamma$, then $i_s^* > i_r^*$. The first order condition then implies $\Gamma_R > 0$. Intuitively, it is not possible to keep banks satiated in reserves when they are being charged for their reserve holdings. More explicitly, we assume that the interest rate on reserves follows a Taylor rule given by equation (9). Because of the reserve management policy outlined above, the deposit rate in equilibrium is either equal to the reserve rate or to the lower bound, as specified in equation (10).

$$i_r^* = r_t^n \Pi_t^{\phi_s} Y_t^{\phi_Y}$$

Another way to interpret the implied link between bank profits and credit supply is to include a capital requirement. In Gerali, Neri, Sessa, and Signoretti (2010) a reduction in bank profits reduces the banks’ capital ratio. In order to recapitalize the bank lowers credit supply. Alternatively, we could assume that bank profits do not affect intermediation costs, but that bank profits are distributed to all households. A reduction in bank profits would then reduce aggregate demand through the borrowers budget constraint.

This policy regime is consistent with a “floor-system” of monetary policy, which has been implemented in many countries, including the U.S, after the crisis. Our model allows us to also analyze a pre-crisis policy regime, in which central banks pay no interest on reserves, so that $i_r^* = 0$. Under such a regime, the policy maker then chooses central bank currency so as to ensure that the risk-free rate is equal to its target.
\[ i_t^* = \max \{ i_t^*, i_t^r \} \]  

(10)

### 3.2 A generalization of the standard New Keynesian model

We take a log-linear approximation of the equilibrium conditions around the steady state, with details outlined in the appendix. The steady state equations, as well as the log-linearized equilibrium conditions are summarized in Appendix C. Here we reproduce the key equations in order to make the following observation: in the absence of interest rate bounds, and any shocks that create a trade-off between inflation and output, our model reduces to the standard New Keynesian model. The central bank can replicate a zero inflation target and keep output at its natural level at all times. Our log-linearized model is therefore a natural generalization of the textbook New Keynesian model with an endogenous natural rate of interest. Our extension, however, goes beyond simply introducing a ZLB along with an endogenous natural rate of interest, as we will have a clear distinction between interest rates on reserves, deposit rates as well as borrowing rates. And while there is no bound on the reserve rate, the deposit rate is subject to a lower bound.

The supply side of our model can be summarized by the generic Phillips curve

\[ \hat{\pi}_t = \kappa \hat{y}_t + \beta E_t \hat{\pi}_{t+1} \]  

(11)

The demand side is governed by the IS-curve

\[ \hat{y}_t = E_t \hat{y}_{t+1} - \sigma \left( i_t^* - E_t \hat{\pi}_{t+1} - \hat{r}_t^n \right) \]  

(12)

In the standard model, the natural rate of interest \( r_t^n \) is exogenous. In our case the natural rate of interest is endogenous, and depends on the shocks to the economy and the agents’ decisions. Specifically, letting \( \hat{\zeta}_t \) denote exogenous shifts in the agents preferred time allocation of consumption, the natural rate of interest takes the following form

\[ \hat{r}_t^n = \hat{\zeta}_t - E_t \hat{\zeta}_{t+1} - \chi \hat{\omega}_t \]  

(13)

The natural rate of interest now depends on the interest rate spread \( \hat{\omega} \), which is endogenous and given by equation (14). Given the assumed functional form for the banks intermediation cost \( \Gamma \) outlined in Appendix D, the link between household debt and the marginal cost of lending is parameterized by \( \nu > 1 \). Moreover, the feedback effect from bank profits to the marginal intermediation cost is captured by the parameter \( \iota \geq 0 \). An increase in private debt increases the interest rate spread, thereby reducing the natural rate of interest. Similarly, a
reduction in bank profits reduces the natural rate of interest if $i > 0$.

$$\hat{\omega}_t = \frac{i^b - i^s}{1 + \hat{\beta}} \left( (\nu - 1)\hat{b}_t - \nu \hat{B}_t - i \hat{z}_t \right)$$

(14)

While the standard New Keynesian model only has one interest rate, our model has three distinct interest rates. We have an interest rate on borrowing $i^b_t$, as well as an interest rate on savings $i^s_t$. In addition we also have an interest rate on central bank reserves $i^r_t$. The log-linearized reserve rate is set by the central bank according to a standard Taylor rule

$$\hat{i}^r_t = \hat{r}^n_t + \phi_\pi \hat{\pi}_t + \phi_y \hat{\pi}_t $$

(15)

Because the central bank keeps the bank sector satiated in reserves whenever feasible, the deposit rate is equal to the reserve rate when the lower bound is not binding, and is equal to the lower bound otherwise

$$\hat{i}^s_t = \max \left\{ \hat{i}^s, \hat{i}^r_t \right\}$$

(16)

where $\hat{i}^s = \frac{\beta_s}{\pi} (1 - \gamma) - 1$ is the lower bound on the deposit rate expressed in deviations from steady state. The current characterization of the model, given by equations (11) - (16) is incomplete, but we relegate to the appendix the full set of equations needed to describe the dynamics of private debt and bank profits in order to conserve space. This partial representation is however sufficient to highlight the important economic mechanisms of our model. If there is no constraint on the policy rate and we set $i^s_t = i^r_t$, then the central bank can fully offset any variations in $r^n_t$ via the policy rate. In this case, as in the standard model, it is easy to confirm that there is an equilibrium in which $\hat{i}^s = \hat{i}^r = \hat{r}^n_t$ and $\hat{\pi}_t = \hat{\pi} = 0$, and the model reduces to the standard model.

3.3 Monetary policy transmission with and without negative interest rates

The characterization above also clarifies how our model provides additional details on the transmission of monetary policy via the banking system. If the central bank lowers the reserve rate in normal times, this lowers the deposit rate through equation (16). The reduction in the deposit rate stimulates the consumption of saver households. In addition, lowering the deposit rate reduces the banks financing costs. This increases their willingness to lend, putting downwards pressure on the borrowing rate and thereby stimulating the consumption of borrower households. Hence, the reduction in the reserve rate passes through to the other interest rates in the economy, thereby stimulating aggregate demand.
Most macro models either implicitly or explicitly assume that the economy has only one interest rate, which the central bank controls directly. This seems like an acceptable assumption in normal times. However, once the policy rate falls below the bound on the deposit rate, allowing for multiple interest rates becomes crucial. As evident from equation (16), lowering the reserve rate below the bound on the deposit rate and into negative territory has no effect on the deposit rate. Further, because the deposit rate stays unchanged, there is no stimulative effect on banks’ financing costs and so no increase in their willingness to lend. As a result, there is no longer a boost to aggregate demand. Moreover, because charging a negative interest rate on reserves reduces bank profits, the interest rate spread in equation (14) increases if $\iota > 0$. This implies an increase in the borrowing rate, and so aggregate demand falls. Hence, when the deposit rate is stuck at its lower bound, and there is a feedback effect from bank profits to the marginal cost of lending, further reductions in the reserve rate have a contractionary effect on the economy. If $\iota = 0$ on the other hand, negative interest rates are neither expansionary nor contractionary. We now illustrate these results with a numerical example.

4 The Effects of Monetary Policy in Positive and Negative Territory

In this section, we compare our baseline model (which we refer to as the negative reserve rates model) to two other models. The first is our variant of the standard lower bound NK model. In this case, there is an identical effective lower bound on both the deposit rate and the central bank’s policy rate. The second is the frictionless model, in which both the deposit rate and the central bank policy rate can fall below zero.

To analyze the dynamic transition of the three models we consider a temporary preference shock - a standard ZLB shock dating at least back to Eggertsson and Woodford (2003). The preference shock effectively makes agents more patient and so delays consumption. We then evaluate to which degree the central bank can stimulate aggregate demand by lowering the reserve rate. In Appendix D we consider an alternative shock, namely a permanent reduction in the debt limit - otherwise known as a debt deleveraging shock (Eggertsson and Krugman, 2012). The qualitative implications for the effectiveness of negative interest rates do not depend on which shock we consider. We pick the size of both shocks to generate an approximate 4.5 percent drop in output on impact. This reduction in output is chosen to roughly mimic the average reduction in real GDP in Sweden, Denmark, Switzerland and the
Euro Area in the aftermath of the financial crisis, as illustrated in Figure 15 in the Appendix.\textsuperscript{21} The drop in output in the US was of similar order. The persistence of the preference shock is set to generate a duration of the lower bound of approximately 12 quarters. We choose the parameters of the model from the existing literature whenever possible. A discussion of the calibration is included in Appendix D.

4.1 Preference shock

The effects of the preference shock in the three different model regimes are depicted in Figure 14.

We start by considering the completely frictionless case, referred to as the No bound case. In this scenario it is assumed that both the policy rate and the deposit rate can turn negative, as illustrated by the dashed black lines in Figure 14.

The preference shock reduces aggregate demand and inflation, triggering an immediate response from the central bank. In the absence of bounds the central bank can hold banks satiated in reserves, and so the reduction in the reserve rate leads to an identical reduction in the deposit rate. The reduction in the deposit rate reduces banks’ financing costs, increases lending and thereby stimulating the consumption of both borrowers and savers.

Contrast the frictionless case to the standard case, in which both the policy rate and the deposit rate are bounded. In this case, the central bank is not able to offset the shock, and output falls below its steady state value. This scenario is outlined by the solid black lines in Figure 14. The key reason is that the central bank is unable to reduce the deposit rate below its lower bound and hence also increase lending. Thus, the expansionary effect on borrowers and savers are muted relative to the frictionless case.

Finally, we consider the case deemed to be most relevant to what we see in the data. While the policy rate is not bounded, there exists an effective lower bound on the deposit rate. This case is illustrated by the red dashed lines in Figure 14.

\textsuperscript{21} Detrended real GDP fell sharply from 2008 to 2009, before partially recovering in 2010 and 2011. The partial recovery was sufficiently strong to induce an interest rate increase. We focus on the second period of falling real GDP (which occurred after 2011), as negative interest rates were not implemented until 2014-2015. Targeting a reduction in real GDP of 4.5 percent is especially appropriate for the Euro Area and Sweden. Real GDP fell by somewhat less in Denmark, and considerably less in Switzerland. This is consistent with the central banks in the Euro Area and Sweden implementing negative rates because of weak economic activity, and the central banks in Denmark and Switzerland implementing negative rates to stabilize their exchange rates.
Figure 14: Impulse response functions following an exogenous decrease in the marginal utility of consumption (\( \zeta \)), under three different models. *Standard model* refers to the case where there is an effective lower bound on both deposit rates and the central bank’s policy rate. *No bound* refers to the case where there is no effective lower bound on any interest-rate. *Negative rates* refers to the model outlined above, where there is an effective lower bound on the deposit rate but no lower bound on the policy rate.

The central bank reacts to the shock by aggressively reducing the policy rate\(^{22}\). However, the deposit rate only responds until it reaches its lower bound, at which point it is stuck. As a result, the borrowing rate does not fall as much as in the frictionless case, and the central bank is once again unable to mitigate the negative effects of the shock on aggregate demand and inflation.

Output declines more in the negative scenario compared to the standard case. The reason is the negative effect on bank profits resulting from the negative interest rate on reserves. Banks hold reserves in order to reduce their intermediation cost, but when the policy rate is negative they are being charged for doing so. At the same time, their financing costs are unresponsive due to the lower bound on the deposit rate. Hence, bank profits are lower when the policy rate is negative\(^{23}\). If \( \tau > 0 \), this decline in bank profits feeds back into aggregate

\(^{22}\)This reaction is exaggerated by our assumption that the central bank literally follows the Taylor rule in setting the interest rate on reserves, while in practice central banks only experimented with modestly negative rates.

\(^{23}\)The fall in bank profits is large in our case, and profits become negative in the negative reserve rate
demand through the effect of bank net worth on the marginal lending cost. Lower net worth increases the cost of financial intermediation, which reduces credit supply and dampens the pass-through of the policy rate to banks’ lending rates.

The importance of profits for banks intermediation costs is parameterized by $\iota$. In Table 3 we report the effect on output and the borrowing rate for different assumptions about $\iota$. In the case in which there is no feedback from bank profits to intermediation costs ($\iota = 0$), the output drop under negative rates corresponds to the output drop under the standard model. The same holds for the borrowing rate. As $\iota$ increases, the reduction in the borrowing rate is muted due to the increase in intermediation costs. As a result, output drops by more. For a sufficiently high $\iota$, the borrowing rate actually increases when negative policy rates are introduced. This is consistent with the bank-level data on daily interest rates from Sweden, where some banks in fact increased their lending rate following the introduction of negative interest rates. Bech and Malkhozov (2016) and Basten and Mariathasan (2018) report a similar increase in lending rates in Switzerland.

<table>
<thead>
<tr>
<th>Model</th>
<th>Output, % deviation from SS</th>
<th>Borrowing rate, percentage points change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>- 4.5</td>
<td>- 3.5</td>
</tr>
<tr>
<td>$\iota = 0$</td>
<td>- 4.5</td>
<td>- 3.5</td>
</tr>
<tr>
<td>$\iota = 0.1$</td>
<td>- 4.7</td>
<td>- 3.3</td>
</tr>
<tr>
<td>$\iota = 0.15$</td>
<td>- 4.8</td>
<td>- 3.1</td>
</tr>
<tr>
<td>$\iota = 0.2$</td>
<td>- 4.9</td>
<td>- 2.9</td>
</tr>
<tr>
<td>$\iota = 0.25$</td>
<td>- 5.0</td>
<td>- 2.7</td>
</tr>
<tr>
<td>$\iota = 0.5$</td>
<td>- 5.7</td>
<td>- 1.1</td>
</tr>
<tr>
<td>$\iota = 0.7$</td>
<td>- 6.9</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Table 3: The effect of a preference shock on output and the borrowing rate (on impact) with negative policy rates for different values of $\iota$. The first row reports the outcomes under the standard model for comparison.

To summarize, negative interest rates are not expansionary relative to the case in which the policy rate is not set below zero. In fact, when $\iota > 0$ there is an additional dampening effect on aggregate demand, making negative interest rates contractionary.

5 Discussion

In our model exercise, the storage cost of money was held fix. However, one could allow the storage cost of money to depend on policy. To the extent that the presence of money is the model. The reason is that the central bank follows the Taylor rule and sets the reserve rate equal to - 17 % in an unsuccessful effort to mitigate the shock. This is an extreme policy compared to the recent experience with negative interest rates, in which interest rates are only modestly below zero.
driving force behind the observed lower bound on deposit rates, affecting the storage cost of money can affect the central bank’s ability to stimulate aggregate demand. There are several ways in which this can be done. The oldest example is a tax on currency, as outlined by Gesell (1916). Gesell’s idea would show up as a direct reduction in the bound on the deposit rate in our model, thus giving the central bank more room to lower the interest rate on reserves and the funding costs of banks. Another possibility is to ban higher denomination bills, a proposal discussed in among others Rogoff (2017c). To the extent that this would increase the storage cost of money, this too, should reduce the bound on the banks deposit rate.

An even more radical idea, which would require some extensions to our model, is to let the reserve currency and the paper currency trade at different values. This proposal would imply an exchange rate between electronic money and paper money, and is discussed in Agarwal and Kimball (2015), Rogoff (2017a) and Rogoff (2017b). A key pillar of the proposal – but perhaps also a challenge to implementability – is that it is the reserve currency which is the economy wide unit of account by which taxes are paid, and accordingly what matters for firm price setting. If such an institutional arrangement is achieved, then there is nothing that prevents a negative interest rate on the reserve currency while cash in circulation would be traded at a different price, given by an arbitrage condition. The take-away from the paper should not be that negative nominal rate are always non-expansionary, simply that they are likely to be so under the current institutional arrangement. This gives all the more reason to contemplate departures from the current framework, such as those mentioned briefly here and discussed in detail by the given authors.

We conclude this paper by discussing some arguments put forward by proponents of negative interest rates. Addressing all of these arguments formally would require expanding our model substantially. Instead, we elaborate informally on whether we believe any of these changes would alter our main conclusions.

**Bank Fees** While banks have largely been unwilling to lower their deposit rates below zero, there has been some discussion surrounding their ability to make up for this by increasing fees and commission income. In our model there are no fixed costs involved in opening a deposit account, but allowing for this would imply that the interest rate on deposits could exceed the effective return on deposits. If banks respond to negative policy rates by increasing their fees, this could in principle reduce the effective deposit rate, and thereby lower banks’ funding costs. However, as illustrated in Figure 10, the commission income of Swedish banks as a share of total assets actually fell after the policy rate turned negative. Hence, the aggregate data does not support the claim that the effective deposit rate is in fact falling. Given that depositors understand that higher fees reduce the effective return on their savings, this is
perhaps not surprising. In any event, the ability to store money should ultimately put a bound on the banks ability to impose fees.

**Alternative Transmission Mechanisms**  Our focus is on the effect of monetary policy on bank interest rates. When the policy rate is positive, lowering it reduces the interest rates charged by banks, which increases credit supply and thereby economic activity. We have shown that this mechanism is weakened once the policy rate turns negative. However, it is still possible that negative interest rates stimulate aggregate demand through other channels. Both the Swiss and Danish central banks motivated their decision to implement negative central bank rates by a need to stabilize the exchange rate. This open economy dimension of monetary policy is absent from our analysis. The point we want to make is that the pass-through to bank interest rates - traditionally the most important channel of monetary policy - is not robust to introducing negative policy rates.

Even if lending volumes do not respond to negative policy rates, there could potentially be an effect on the composition of borrowers. It has been suggested that banks may respond to negative interest rates by increasing risk taking. This could potentially increase lending rates, resulting in upward pressure on the interest rate margin. Heider, Saidi, and Schepens (2016) find support for increased risk taking in the Euro Area, using volatility in the return-to-asset ratio as a proxy for risk taking. According to their results, banks in the Euro Area responded to the negative policy rate by increasing return volatility. This is certainly not the traditional transmission mechanism of monetary policy, and it is unclear whether such an outcome is desirable.

**Other policies**  Our model exercise focuses exclusively on the impact of negative policy rates. Other monetary policy measures which occurred over the same time period are not taken into account. This is perhaps especially important to note in the case of the ECB, which implemented its targeted longer-term refinancing operations (TLTROs) simultaneously with lowering the policy rate below zero. Under the TLTRO program, banks can borrow from the ECB at attractive conditions. Both the loan amount and the interest rate are tied to the banks’ loan provision to households and firms. The borrowing rate can potentially be as low as the interest rate on the deposit facility, which is currently -0.40 percent. Such

---

24It is worth mentioning that in our model all reserves earn the same interest rate. In reality, most central banks have implemented a tiered remuneration scheme, in which case the marginal and average reserve rates differ. For example, some amount of reserves may pay a zero interest rate, while reserves in excess of this level earn a negative rate. We outline the policy schemes in the different countries in appendix C. Allowing for more than one interest rate on reserves would not qualitatively alter our results, but would be relevant for a more detailed quantitative assessment.

25In our model we only consider a negative interest rate on bank assets, as we impose $R \geq 0$. The TLTRO program implies a negative interest rate on a bank liability.
a subsidy to bank lending is likely to affect both bank interest rates and bank profits, and could potentially explain why lending rates in the Euro Area have fallen more than in other places once the policy rate turned negative.

6 Summary

Since 2014, several countries have experimented with negative policy rates. In this paper, we have documented that negative central bank rates have not been transmitted to aggregate deposit rates, which remain stuck at levels close to zero. As a result, aggregate lending rates remain elevated as well. Using bank level data from Sweden, we documented a disconnect between the policy rate and lending rates, once the policy rate fell below zero. We further showed that this disconnect was partially explained by reliance on deposit financing. Consistent with this, we found that Swedish banks with high deposit shares cut back on lending relative to other banks - once the policy rate turned negative.

Motivated by our empirical findings, we developed a New Keynesian model with savers, borrowers, and a bank sector. By including money storage costs and central bank reserves, we captured the disconnect between the policy rate and the deposit rate at the lower bound. In this framework we showed that a negative policy rate was at best irrelevant, but could potentially be contractionary due to a negative effect on bank profits.

Given the long-term decline in interest rates, the need for unconventional monetary policy is likely to remain high in the future. Our findings suggest that negative interest rates are not an effective tool in fighting off the next recession. The questions remains however, what is? Alternative monetary policy measures include quantitative easing, forward guidance and credit subsidies such as the TLTRO program implemented by the ECB. While existing literature has made progress in evaluating these measures, the question of how monetary policy should optimally be implemented in a low interest rate environment remains largely unresolved.
References


A Additional Figures

Figure 15: Gross domestic product in constant prices. Local currency. Indexed so that GDP\textsuperscript{2008}=100. The right panel shows the detrended series using a linear time trend based on the 1995-2007 period.

Figure 16: Estimated average funding costs. The estimated average funding cost is computed by taking the weighted average of the assumed interest rate of the different funding sources of the bank. Certificates are assumed to have the same interest rate as STIBOR 3M, while unsecured debt are assumed to have the same interest rate as STIBOR 3M plus a 2 percent constant risk-premium. The “counterfactual” series correspond to the case when the spread between the repo rate and the estimated funding cost remain fixed at pre-negative levels. Weights from Figure 11 used.
Figure 17: Comparing bank level mortgage rates to aggregate data. We aggregate the bank level mortgage rates using market shares from the Swedish Banker’s Association (2017), supplemented with data on lending volumes from Statistics Sweden. The blue line (Aggregate Data) depicts the official average mortgage interest rate for loans with a fixed interest rate period of 3-5 years from Statistics Sweden.

B Marginal and Average Rate on Reserves

In our model, central bank reserves earn a single interest rate \( i^r \). In reality, central banks can adopt exemption thresholds and tiered remuneration schemes so that not all reserves earn the same interest rate. Hence, even though the key policy rate is negative, not all central bank reserves necessarily earn a negative interest rate. Here we provide a short overview of the different remuneration schedules implemented in the Euro Area, Denmark, Japan, Sweden and Switzerland. For a more detailed analysis see Bech and Malkhozov (2016).

In the Euro Area, required reserves earn the main financing operations rate - currently set at 0.00 percent. Excess reserves on the other hand, earn the central bank deposit rate - currently set at -0.40 percent. Hence, only reserves in excess of the required level earn a negative interest rate. A similar remuneration scheme is in place in Denmark. Banks can deposit funds at the Danish central bank at the current account rate of 0.00 percent. However, there are (bank-specific) limits on the amount of funds that banks can deposit at the current account rate. Funds in excess of these limits earn the interest rate on one-week certificates of deposits - currently set at -0.65 percent.

The Riksbank issues one-week debt certificates, which currently earn an interest rate of -0.50 percent. While there is no reserve requirement, the Swedish central bank undertakes fine-tuning operations to drain the bank sector of remaining reserves each day. These fine-tuning operations earn an interest rate of -0.60 percent\(^{26}\). The Swiss central bank has the

\(^{26}\)Any residual reserves earn the deposit rate of -1.25 percent.
lowest key policy rate at -0.75 percent. However, due to high exemption thresholds the majority of reserves earn a zero interest rate. The Bank of Japan adopted a three-tiered remuneration schedule when the key policy rate turned negative. As a result, central bank reserves earn an interest rate of either 0.10, 0.00 or -0.10 percent.

Due to the tiered remuneration system, there is a gap between the average and the marginal reserve rate. Bech and Malkhozov (2016) calculate this gap as of February 2016, as illustrated in Figure 18.

<table>
<thead>
<tr>
<th>Central bank remuneration schedules (mid-February 2016)</th>
<th>Table 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exemption threshold</td>
<td>European Central Bank</td>
</tr>
<tr>
<td>Minimum reserve requirement</td>
<td></td>
</tr>
<tr>
<td>Aggregate amounts</td>
<td>Local currency, in billions</td>
</tr>
<tr>
<td>Overnight deposits (reserves)</td>
<td></td>
</tr>
<tr>
<td>Below threshold</td>
<td>113</td>
</tr>
<tr>
<td>Above threshold</td>
<td>650</td>
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<tr>
<td>Term (one-week)</td>
<td></td>
</tr>
<tr>
<td>Policy rates</td>
<td>Basis points</td>
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<tr>
<td>Overnight deposits (reserves)</td>
<td></td>
</tr>
<tr>
<td>Below threshold</td>
<td>5</td>
</tr>
<tr>
<td>Above threshold</td>
<td>-30</td>
</tr>
<tr>
<td>Term (one-week)</td>
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</tr>
<tr>
<td>Weighted average rate</td>
<td>-25</td>
</tr>
<tr>
<td>Marginal minus average rate*</td>
<td>-5</td>
</tr>
</tbody>
</table>

1 Amount of fine-tuning operations. In addition, overnight deposits with central bank represent SEK 0.01 billion. 2 Rate applied to fine-tuning operations. Overnight deposits with central bank earn -125 basis points. 3 Amounts above the aggregate current account limit are converted into one-week certificates of deposit (Box 2). 4 Marginal rate is the rate on overnight deposits with central bank above exemption threshold.

Sources: Central banks; authors’ calculations.

Figure 18: Reserve rates - Source: Bech and Malkhozov (2016).

C Details of the model

Households

We consider a closed economy, populated by a unit-measure continuum of households. Households are of two types, either patient (indexed by superscript s) or impatient (indexed by superscript b). Patient households have a higher discount factor than impatient agents, i.e. $\beta^s > \beta^b$. The total mass of patient households is $1 - \chi$, while the total mass of impatient households is $\chi$. In equilibrium, impatient households will borrow from patient households via the banking system, which we specify below. We therefore refer to the impatient households...
as “borrowers” and the patient households as “savers”.

Households consume, supply labor, borrow/save and hold real money balances. At any time \( t \), the optimal choice of consumption, labor, borrowing/saving and money holdings for a household \( j \in \{s, b\} \) maximizes the present value of the sum of utilities

\[
U^j_t = E_t \sum_{T=t}^{\infty} (\beta^j)^{T-t} \left[ U \left( C^j_T \right) + \Omega \left( \frac{M^j_T}{P_T} \right) - V \left( N^j_T \right) \right] \zeta_t
\]

where \( \zeta_t \) is a random variable following some stochastic process and acts as a preference shock\(^{27}\). \( C^j_t \) and \( N^j_t \) denote consumption and labor for type \( j \) respectively, and the utility function satisfies standard assumptions clarified below.

Households consume a bundle of consumption goods. Specifically, there is a continuum of goods indexed by \( i \), and each household \( j \) has preferences over the consumption index

\[
C^j_t = \left( \int_0^1 C_t \left( i \right)^{\frac{\theta - 1}{\theta}} di \right)^{\frac{\theta}{\theta - 1}}
\]

where \( \theta > 1 \) measures the elasticity of substitution between goods.

Agents maximize lifetime utility (equation (17)) subject to the following flow budget constraint:

\[
M^j_t + B^j_{t-1} (1 + i^j_{t-1}) = W^j_t N^j_t + B^j_t + M^j_{t-1} - P_t C^j_t - S \left( M^j_{t-1} \right) + \Psi^j_t + \psi^j_t - T^j_t
\]

\( B^j_t \) denotes one period risk-free debt of type \( j \) (\( B^s_t < 0 \) and \( B^b_t > 0 \)). For the saver, \( B^s_t \) consists of bank deposits and government bonds, both remunerated at the same interest rate \( i^s_t \) by arbitrage. Borrower households borrow from the bank sector only, at the banks’ lending rate \( i^b_t \). \( S \left( M^j_{t-1} \right) \) denotes the storage cost of holding money. \( \Psi^j_t \) is type \( j \)'s share of firm profits, and \( \psi^j_t \) is type \( j \)'s share of bank profits. Let \( Z^\text{firm}_t \) denote firm profits, and \( Z_t \) denote bank profits. We assume that firm profits are distributed to both household types based on their population shares, i.e. \( \Psi^b_t = \chi Z^\text{firm}_t \) and \( \Psi^s_t = (1 - \chi) Z^\text{firm}_t \). Bank profits on the other hand are only distributed to savers, which own the deposits by which banks finance themselves\(^{28}\). Hence, we have that \( \psi^b_t = 0 \) and \( \psi^s_t = Z_t \).

\(^{27}\)We introduce the preference shock as a parsimonious way of engineering a recession.

\(^{28}\)Distributing bank profits to both household types would make negative interest rates even more contractionary. The reduction in bank profits would reduce the transfer income of borrower households, causing them to reduce consumption. We believe this effect to be of second order significance, and so we abstract from it here.
The optimal consumption path for an individual of type $j$ has to satisfy the standard Euler-equation

$$U' \left( C^j_t \right) \zeta_t = \beta^j \left( 1 + \bar{i}^j_t \right) \mathbb{E}_t \left( \Pi_{t+1}^{-1} U' \left( C^j_{t+1} \right) \zeta_{t+1} \right)$$ \hspace{1cm} (20)

Optimal labor supply has to satisfy the intratemporal trade-off between consumption and labor\footnote{We assume that the function $V$ is increasing in $N$ and convex with well defined first and second derivatives.}

$$\frac{V' \left( N^j_t \right)}{U' \left( C^j_t \right)} = \frac{W^j_t}{P_t}$$ \hspace{1cm} (21)

Finally, optimal holdings of money have to satisfy\footnote{We assume a satiation point for money. That is, at some level $\bar{m}^j$ households become satiated in real money balances, and so $\Omega' \left( \bar{m}^j \right) = 0$.}

$$\frac{\Omega' \left( \frac{M^j_t}{P_t} \right)}{U' \left( C^j_t \right)} = \frac{i^j_t + S' \left( M^j_t \right)}{1 + \bar{i}^j_t}$$ \hspace{1cm} (22)

The lower bound on the deposit rate $\bar{i}^s$ is typically defined as the lowest value of $i^s_t$ satisfying equation (22). The lower bound therefore depends crucially on the marginal storage cost. With the existence of a satiation point in real money balances, zero (or constant) storage costs imply $S' \left( M^s_t \right) = 0$ and $\bar{i}^s = 0$. That is, the deposit rate is bounded at exactly zero. With a non-zero marginal storage cost however, this is no longer the case. If storage cost are convex, for instance, the marginal storage cost is increasing in $M^s_t$. In this case, there is no lower bound. Based on the data from section (2), a reasonable assumption is that the deposit rate is bounded at some value close to zero. This is consistent with a proportional storage cost $S \left( M^s_t \right) = \gamma M^s_t$, with a small $\gamma > 0$. We therefore assume proportional storage costs for the rest of the paper, in which the lower bound on deposit rates is given by $\bar{i}^s = -\gamma$\footnote{This nests the case of no storage costs, in which case $\gamma = 0$.}

We assume that households have exponential preferences over consumption, i.e. $U(C^j_t) = 1 - exp \left\{ -qC^j_t \right\}$ for some $q > 0$. The assumption of exponential utility is made for simplicity, as it facilitates aggregation across agents. Under these assumptions, the labor-consumption trade-off can easily be aggregated into an economy-wide labor market condition\footnote{To see this, just take the weighted average of equation (21) using the population shares $\chi$ and $1 - \chi$ as the respective weights.}

$$\frac{V' \left( N_t \right)}{U' \left( C_t \right)} = \frac{W_t}{P_t}$$ \hspace{1cm} (23)
Letting $G_t$ denote government spending\textsuperscript{33}, aggregate demand is given by

$$Y_t = \chi C_t^b + (1 - \chi) C_t^s + G_t \quad (24)$$

**Firms**

Each good $i$ is produced by a firm $i$. Production is linear in labor, i.e.

$$Y_t(i) = N_t(i) \quad (25)$$

where $N_t(i)$ is a Cobb-Douglas composite of labor from borrowers and savers respectively, i.e. $N_t(i) = (N_t^b(i))^\chi (N_t^s(i))^{1-\chi}$, as in Benigno, Eggertsson, and Romei (2014). This ensures that each type of labor receives a total compensation equal to a fixed share of total labor expenses. That is,

$$W_t^b N_t^b = \chi W_t N_t \quad (26)$$
$$W_t^s N_t^s = (1 - \chi) W_t N_t \quad (27)$$

where $W_t = (W_t^b)^\chi (W_t^s)^{1-\chi}$ and $N_t = \int_0^1 N_t(i) \, di$.

Given preferences, firms face a downward-sloping demand function

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^\theta Y_t \quad (28)$$

We introduce nominal rigidities by assuming Calvo-pricing. That is, in each period, a fraction $\alpha$ of firms are not able to reset their price. Thus, the likelihood that a price set in period $t$ applies in period $T > t$ is $\alpha^{T-t}$. Prices are assumed to be indexed to the inflation target $\Pi$.

A firm that is allowed to reset their price in period $t$ sets the price to maximize the present value of discounted profits in the event that the price remains fixed. That is, each firm $i$ choose $P_t(i)$ to maximize

$$\mathbb{E}_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} \lambda_T \left[ \Pi^{T-t} \frac{P_t(i)}{P_T} Y_T(i) - \frac{W_T}{P_T} Y_T(i) \right] \quad (29)$$

\textsuperscript{33}Government policies are explained below.
where $\lambda_T \equiv q(\chi \exp \{-qC^b_T\} + (1 - \chi) \exp \{-qC^s_T\})$, which is the weighted marginal utility of consumption and $\beta \equiv \chi \beta^b + (1 - \chi) \beta^s$.

Denoting the markup as $\mu \equiv \theta / (\theta - 1)$, firms set the price as a markup over the average of expected marginal costs during the periods the price is expected to remain in place. That is, the first-order condition for the optimal price $P(i)^*_t$ for firm $i$ is

$$
\frac{P(i)^*_t}{P_t} = \mu \frac{\mathbb{E}_t \left\{ \sum_{T=1}^\infty (\alpha \beta)^{T-t} \lambda_T \left( \frac{P_T}{P_t} \frac{1}{\Pi_T^{T-t}} \right)^\theta W_T Y_T \right\}}{\mathbb{E}_t \left\{ \sum_{T=1}^\infty (\alpha \beta)^{T-t} \lambda_T \left( \frac{P_T}{P_t} \frac{1}{\Pi_T^{T-t}} \right)^{\theta-1} W_T Y_T \right\}}
$$

(30)

This implies a law of motion for the aggregate price level

$$
P_t^{1-\theta} = (1 - \alpha) P_t^{1-\theta} + \alpha P_t^{1-\theta} \Pi_t^{1-\theta}
$$

(31)

where $P_t^*$ is the optimal price from equation (30), taking into account that in equilibrium $P_t^*(i)$ is identical for all $i$. We denote this price $P_t^*$.

Since prices are sticky, there exists price dispersion which we denote by

$$
\Delta_t \equiv \int_0^1 \left( \frac{P_t^*(i)}{P_t} \right)^{-\theta} \, di
$$

(32)

with the law of motion

$$
\Delta_t = \alpha \left( \frac{\Pi_t}{\Pi} \right)^{\theta} \Delta_{t-1} + (1 - \alpha) \left( 1 - \alpha \left( \frac{\Pi_t}{\Pi} \right)^{\theta-1} \right) \left( \frac{1}{1 - \alpha} \right) \left( \frac{\Pi_t}{\Pi} \right)^{\theta} \pi_t
$$

(33)

We assume that the disutility of labor takes the form $V(N^j_t) = (N^j_t)^{1+\eta}$. We can then combine equations (30) - (33), together with the aggregate labor-consumption trade-off (equation (23)) to get an aggregate Phillips curve of the following form:

$$
\left( 1 - \alpha \left( \frac{\Pi_t}{\Pi} \right)^{\theta-1} \right) \left( \frac{1}{1 - \alpha} \right) \left( \frac{\Pi_t}{\Pi} \right)^{\theta} \pi_t \pi_t = \frac{F_t}{K_t}
$$

(34)

where

34Recall that the firm is owned by both types of households according to their respective population shares.
\[ F_t = \lambda_t Y_t + \alpha \beta \mathcal{E}_t \left\{ F_{t+1} \left( \frac{\Pi_{t+1}}{\Pi} \right)^{\theta-1} \right\} \] (35)

and

\[ K_t = \mu \frac{\lambda_t \Delta Y^{1+\eta}}{z \exp \{-z Y_t\}} + \alpha \beta \mathcal{E}_t \left\{ K_{t+1} \left( \frac{\Pi_{t+1}}{\Pi} \right)^{\theta} \right\} \] (36)

and

\[ \lambda_T \equiv z \left( \chi \exp \{-q C^b_T\} + (1 - \chi) \exp \{-q C^s_T\} \right) \] (37)

Since every firm faces demand \( Y(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\theta} \) \( Y_t \) and \( Y_t(i) = N_t(i) \), we can integrate over all firms to get that

\[ N_t = \Delta_t Y_t \] (38)

**Banks**

Our banking sector is made up of identical, perfectly competitive banks. Bank assets consist of one-period real loans \( l_t \). In addition to loans, banks hold real reserves \( R_t \geq 0 \) and real money balances \( m_t = M_t P_t \geq 0 \), both issued by the central bank\(^{35} \). Bank liabilities consist of real deposits \( d_t \). Reserves are remunerated at the interest rate \( i^r \), which is set by the central bank. Loans earn a return \( i^b \). The cost of funds, i.e. the deposit rate, is denoted \( i^s \). Banks take all of these interest-rates as given.

Financial intermediation takes up real resources. Therefore, in equilibrium there is a spread between the deposit rate \( i^s \) and the lending rate \( i^b \). We assume that banks’ intermediation costs are given by a function \( \Gamma \left( \frac{l_t}{l_t}, R_t, m_t, z_t \right) \), in which \( z_t = \frac{Z_t}{P_t} \) is real bank profit. In order to allow for the intermediation cost to be time-varying for a given set of bank characteristics, we include a stochastic cost-shifter \( l_t \). This cost-shifter may capture time-variation in borrowers default probabilities, changes in borrower households borrowing capacity, bank regulation etc. (Benigno, Eggertsson, and Romei 2014).

We assume that the intermediation costs are increasing and convex in the amount of real loans provided. That is, \( \Gamma_l > 0 \) and \( \Gamma_{ll} \geq 0 \). Central bank currency plays a key role in reducing intermediation costs\(^{36} \). The marginal cost reductions from holding reserves and

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\(^{35}\)Because we treat the bank problem as static - as outlined below - we can express the maximization problem in real terms.

\(^{36}\)For example, we can think about this as capturing in a reduced form way the liquidity risk that banks face. When banks provide loans, they take on costly liquidity risk because the deposits created when the
money are captured by $\Gamma_R \leq 0$ and $\Gamma_m \leq 0$ respectively. We assume that the bank becomes satiated in reserves for some level $\bar{R}$. That is, $\Gamma_R = 0$ for $R \geq \bar{R}$. Similarly, banks become satiated in money at some level $\bar{m}$, so that $\Gamma_m = 0$ for $m \geq \bar{m}$. Banks can thus reduce their intermediation costs by holding reserves and/or cash, but the opportunity for cost reduction can be exhausted. Finally, we assume that higher profits (weakly) reduce the marginal cost of lending. That is, we assume $\Gamma_{l_z} \leq 0$. We discuss this assumption below.

Following Curdia and Woodford (2011) and Benigno, Eggertsson, and Romei (2014) we assume that any real profits from the bank’s asset holdings are distributed to their owners in period $t$ and that the bank holds exactly enough assets at the end of the period to pay off the depositors in period $t+1$. Furthermore, we assume that storage costs of money are proportional and given by $S(M) = \gamma M$. Under these assumptions, real bank profits can be implicitly expressed as:

$$z_t = \frac{i_t^b - i_t^s}{1 + i_t^s} l_t - \frac{i_t^s - i_t^s}{1 + i_t^s} R_t - \frac{i_t^s + \gamma}{1 + i_t^s} m_t - \Gamma \left( \frac{l_t}{l_t}, R_t, m_t, z_t \right)$$

Any interior $l_t$, $R_t$ and $m_t$ have to satisfy the respective first-order conditions from the bank’s optimization problem:

$$l_t : \frac{i_t^b - i_t^s}{1 + i_t^s} = \frac{1}{l_t} \Gamma_l \left( \frac{l_t}{l_t}, R_t, m_t, z_t \right)$$

$$R_t : -\Gamma_R \left( \frac{l_t}{l_t}, R_t, m_t, z_t \right) = \frac{i_t^s - i_t^s}{1 + i_t^s}$$

$$m_t : -\Gamma_m \left( \frac{l_t}{l_t}, R_t, m_t, z_t \right) = \frac{i_t^s + \gamma}{1 + i_t^s}$$

The first-order condition for real loans says that the banks trade off the marginal profits from lending with the marginal increase in intermediation costs. The next two first-order conditions describe banks demand for reserves and cash. We assume that reserves and money are not perfect substitutes, and so minimizing the intermediation cost implies holding both reserves and money. This is not important for our main result.

---

37 The latter is equivalent to assuming that $(1 + i_t^b) l_t + (1 + i_t^r) R_t + m_t - S(m_t) = (1 + i_t^s) d_t$.

38 Assuming that $\Gamma \left( \frac{l_t}{l_t}, R_t, m_t, z_t \right)$ is such that there exists a unique $z$ solving equation (39).

39 The assumption that banks always wants to hold some reserves is however important for the effect of negative interest rates on bank profitability. If we instead assume that the sum of money holdings and reserves enters the banks cost function as one argument, the bank would hold only money once $i_t^r < -\gamma$. Hence, reducing the interest rate on reserves further would not affect bank profits. However, such a collapse in
The first-order condition for loans pins down the equilibrium credit spread \( \omega_t \) defined as

\[
\omega_t \equiv \frac{1 + i^b_t}{1 + i^s_t} - 1 = \frac{i^b_t - i^s_t}{1 + i^s_t} \tag{43}
\]

Specifically, it says that

\[
\omega_t = \frac{1}{\chi b_t} \Gamma_l \left( \frac{b^b_t}{b_t}, R_t, m_t, z_t \right) \tag{44}
\]

where we have used the market clearing condition in equation (45) to express the spread as a function of the borrowers real debt holdings \( b^b_t \).

\[
l_t = \chi b^b_t \tag{45}
\]

That is, the difference between the borrowing rate and the deposit rate is an increasing function of the aggregate relative debt level, and a decreasing function of banks’ net worth.

**Why do bank profits affect intermediation costs?** We have assumed that the marginal cost of extending loans (weakly) decreases with bank profits. That is, \( \Gamma_{lz} \leq 0 \). This assumption captures, in a reduced form manner, the established link between banks’ net worth and their operational costs. We do not make an attempt to microfound this assumption here, which is explicitly done in among others Holmstrom and Tirole (1997) and Gertler and Kiyotaki (2010)41.

In Gertler and Kiyotaki (2010) bank managers may divert funds, which means that banks must satisfy an incentive compatibility constraint in order to obtain external financing. This constraint limits the amount of outside funding the bank can obtain based on the banks net worth. Because credit supply is determined by the total amount of internal and external funding, this means that bank lending depends on bank profits. In an early contribution, Holmstrom and Tirole (1997) achieve a similar link between credit supply and bank net worth by giving banks the opportunity to engage (or not engage) in costly monitoring of its non-financial borrowers. For recent empirical evidence on the relevance of bank net worth in explaining credit supply, see for example Jiménez, Ongena, Peydró, and Saurina (2012).

Importantly, our main result is that negative interest rates are not expansionary. This does not depend on profits affecting intermediation costs. However, the link between profits central bank reserves is not consistent with data, suggesting that banks want to hold some (excess) reserves.40Following equation (45) we also assume that \( \tilde{l}_t = \chi b_t \).

41Another way to interpret the implied link between bank profits and credit supply is to include a capital requirement. In Gerali, Neri, Sessa, and Signoretti (2010) a reduction in bank profits reduces the banks’ capital ratio. In order to recapitalize the bank lowers credit supply.
and the intermediation cost is the driving force behind negative interest rates being *contractionary*. If we turn off this mechanism, negative interest rates still reduce bank profits, but this does not feed back into aggregate demand\(^{42}\).

**Policy**

The consolidated government budget constraint is given by

\[
B^g_t + M^{tot}_t + P_t R_t = (1 + i^g_{t-1}) B^g_{t-1} + M^{tot}_{t-1} + (1 + i^r_{t-1}) P_{t-1} R_{t-1} + G_t - T_t
\]  

(46)

where \(B^g_t\) is one period government debt, \(M^{tot}_t = M_t + M^s_t + M^b_t\) is total money supply - which is the sum of money held by banks and each household type, \(i^g_t\) is the one period risk-free rate on government debt, \(G_t\) is government spending, and \(T_t = \chi T^b_t + (1 - \chi) T^s_t\) is the weighted sum of taxes on the two household types.

The conventional way of defining monetary and fiscal policy, abstracting from reserves and the banking sector (see e.g. Woodford 2003), is to say that fiscal policy is the determination of end of period government liabilities, i.e. \(B^g_t + M^{tot}_t\), via the fiscal policy choice of \(G_t\) and \(T_t\). Monetary policy on the other hand, determines the *split* of end of period government liabilities \(B^g_t\) and \(M^{tot}_t\), via open market operations. This in turn determines the risk-free nominal interest rate \(i^g_t\) through the money demand equations of the agents in the economy. The traditional assumption then, is that the one period risk-free rate on government debt corresponds to the policy rate which the monetary authority controls via the supply of money through the money demand equation.

We define monetary and fiscal policy in a similar way here. Fiscal policy is the choice of fiscal spending \(G_t\) and taxes \(T_t\). This choice determines total government liabilities at the end of period \(t\) – the left hand side of equation (46). Total government liabilities are now composed of public debt and the money holdings of each agent, as well as reserves held at the central bank. Again, monetary policy is defined by how total government liabilities is split between government bonds \(B^g_t\), and the overall supply of central bank issuance. In addition, we assume that the central bank sets the interest rate on reserves \(i^r_t\). The supply of central bank currency is then given by

\[
CBC_t = P_t R_t + M_t + M^s_t + M^b_t
\]  

(47)

Given these assumptions, the financial sector itself determines the allocation between re-

\(^{42}\)Alternatively, we could assume that bank profits do not affect intermediation costs, but that bank profits are distributed to all households. A reduction in bank profits would then reduce aggregate demand through the borrowers budget constraint.
serves and money. That is, the split between the money holdings of different agents and reserves held by banks is an endogenous market outcome determined by the first order conditions of banks and households.

In order to clarify the discussion, it is helpful to review two policy regimes observed in the US at different times. Consider first the institutional arrangement in the US prior to the crisis, when the Federal Reserve paid no interest on reserves, so that $i_r = 0$. As seen from equation (41), this implies that banks were not satiated in reserves. The policy maker then chose $CBC_t$ so as to ensure that the risk-free rate was equal to its target. In this more general model, the policy rate is simply the risk-free nominal interest rate, which is equal to the deposit rate and, assuming that depositors can also hold government bonds, the interest rate paid on one period government bonds, i.e. $i_s = i^g_t$.

Consider now an alternative institutional arrangement, in which paying interest on reserves is a policy tool. Such a regime seems like a good description of the post-crisis monetary policy operations, both in the US and elsewhere. The central bank now sets the interest rate on reserves equal to the risk-free rate, i.e. $i_r = i_s = i^g_t$, and chooses $CBC_t$ to implement its desired target. From the first order condition for reserves (41), we see that $i_s = i_r$ implies that $\Gamma_R = 0$. Hence, as long as banks are satiated in reserves, the central bank implicitly controls $i_s$ via $i_r$. A key point, however, is that $\Gamma_R = 0$ is not always feasible due to the lower bound on the deposit rate. If the deposit rate is bounded at $i_s = -\gamma$, and the central bank lowers $i_r$ below $-\gamma$, then $i_s > i_r$. The first order condition then implies $\Gamma_R > 0$. Intuitively, it is not possible to keep banks satiated in reserves when they are being charged for their reserve holdings. More explicitly, we assume that the interest rate on reserves follows a Taylor rule given by equation (48).

Before closing this section it is worth pointing out that it seems exceedingly likely that there also exists a lower bound on the reserve rate. Reserves are useful for banks because they are used to settle cash-balances between banks at the end of each day. However, banks could in principle settle these balances outside of the central bank, for example by ferrying currency from one bank to another (or more realistically trade with a privately owned clearing house.

\begin{equation}
\hat{i}^s_t = r_t^a \Pi_t^s Y_t^{\phi_Y} \tag{48}
\end{equation}

\begin{equation}
i_s^t = \max \{ i^s, i_r^t \} \tag{49}
\end{equation}
where the commercial banks can store cash balances). Hence, because banks have the option to exchange their reserves for cash, there is a limit to how negative $i_t^r$ can become. We do not model this bound here, as it does not appear to have been breached (yet) in practice. Instead we focus on the bound on deposit rates - which is observable in the data.

**Equilibrium**

**Non-linear Equilibrium Conditions**

For given initial conditions $\Delta_0, b_0^b$ and a sequence of shocks $\{\zeta_t, \bar{l}_t\}_{t=0}^{\infty}$, an equilibrium in our model is a sequence of endogenous prices $\{i_t^s, i_t^b, i_t^r\}_{t=0}^{\infty}$ and endogenous variables $\{C_t^b, C_t^s, b_t^b, m_t^b, m_t^s, \tau_t^s, cbc_t, Y_t, \Pi_t, F_t, K_t, \Delta_t, \lambda_t, h_t, R_t, m_t, z_t\}_{t=0}^{\infty}$ such that the 20 equations listed below are satisfied.
\[
\exp \left\{-q C^b_t \right\} \zeta_t = \beta^b \left(1 + i_t^b \right) \mathbb{E}_t \left( \Pi_{t+1}^{-1} \exp \left\{-q C^b_{t+1} \right\} \right) \zeta_{t+1} \\
\exp \left\{-q C^s_t \right\} \zeta_t = \beta^s \left(1 + i_t^s \right) \mathbb{E}_t \left( \Pi_{t+1}^{-1} \exp \left\{-q C^s_{t+1} \right\} \right) \zeta_{t+1} \\
C_t^b + m_t^b + \frac{1 + i_{t-1}^b}{\Pi_t} b_{t-1}^b = \chi Y_t + \frac{1 - \gamma}{\Pi_t} m_{t-1}^b + b_t^b \\
\Omega' \left( m_t^b \right) \left/ \Omega' \left( C_t^b \right) \right. = \frac{i_t^b + \gamma}{1 + i_t^b} \\
\Omega' \left( m_t^s \right) \left/ \Omega' \left( C_t^s \right) \right. = \frac{i_t^s + \gamma}{1 + i_t^s} \\
\Pi_t cbc_t = cbc_{t-1} + i_{t-1}^c R_{t-1} - \Pi_t \tau_t^s \\
cbc_t = R_t + m_t + m_t^s + m_t^b \\
Y_t = \chi C_t^b + (1 - \chi) C_t^s \\
\frac{1 - \alpha \left( \frac{\Pi_t}{\Pi} \right)^{\theta-1}}{1 - \alpha} = \frac{F_t}{K_t} \\
F_t = \lambda_t Y_t + \alpha \beta \mathbb{E}_t \left\{ F_{t+1} \left( \frac{\Pi_{t+1}}{\Pi} \right)^{\theta-1} \right\} \\
K_t = \mu \left( \frac{\lambda_t \Delta_t^\eta Y_t^{1+\eta}}{q \exp \left\{-q Y_t \right\}} + \alpha \beta \mathbb{E}_t \left\{ K_{t+1} \left( \frac{\Pi_{t+1}}{\Pi} \right)^{\theta} \right\} \right) \\
\lambda_t = q \left( \chi \exp \left\{-q C_t^b \right\} + (1 - \chi) \exp \left\{-q C_t^s \right\} \right) \\
\Delta_t = \alpha \left( \frac{\Pi_t}{\Pi} \right)^{\theta} \Delta_{t-1} + (1 - \alpha) \left( \frac{1 - \alpha \left( \frac{\Pi_t}{\Pi} \right)^{\theta-1}}{1 - \alpha} \right)^{\frac{\theta}{\theta-\eta}} \\
z_t^c = \frac{i_t^c - i_t^s}{1 + i_t^s} l_t^c - \frac{i_t^c - i_t^s}{1 + i_t^s} R_t - \frac{i_t^c + \gamma}{1 + i_t^s} m_t - \Gamma \left( \frac{l_t^c}{l_t^c}, R_t, m_t, z_t \right) \\
\frac{i_t^c - i_t^s}{1 + i_t^s} = \frac{1}{l_t^c} \Gamma \left( \frac{l_t^c}{l_t^c}, R_t, m_t, z_t \right) \\
-\Gamma_R \left( \frac{l_t^c}{l_t^c}, R_t, m_t, z_t \right) = \frac{i_t^c - i_t^s}{1 + i_t^s} \\
-\Gamma_m \left( \frac{l_t^c}{l_t^c}, R_t, m_t, z_t \right) = \frac{i_t^c + \gamma}{1 + i_t^s} \\
i_t^c = \max \left\{-\gamma, i_t^c \right\} \\
l_t = \chi b_t^b
\]
Steady state

We denote the steady-state value of a variable $X_t$ as $X$.

First, observe that in steady-state inflation is at the inflation target $\Pi$. As a result, there is no price dispersion ($\Delta = 1$).

Combining this with the Phillips curve, we have that steady-state output is pinned down by the following equation

$$\mu \frac{Y^n}{q \exp \{-qY\}} = 1 \quad (70)$$

From the Euler equation of a household of type $j$ we have that

$$1 + i^j = \frac{\Pi}{\beta^j} \quad (71)$$

Using the steady-state interest rates, we can jointly solve for all bank-variables. Notice that in steady-state banks are satiated in reserves, and so $R = R$ by assumption. Furthermore, if the intermediation cost function is additive between money and the other arguments (which we assume, see below), the steady-state level of money holdings for banks is independent of other bank variables. Therefore, only bank profits and bank lending have to be solved jointly.

Given total debt and interest rates, the borrowers budget constraint and money demand can be solved for steady state consumption and money holdings:

$$C^b = \chi Y + \frac{\Pi - 1 - i^b}{\Pi} b^b - \frac{\Pi - 1 + \gamma}{\Pi} m^b \quad (72)$$

$$\Omega'(m^b) = \frac{i^b + \gamma}{1 + i^b} U''(C^b) \quad (73)$$

Then, using the aggregate resource constraint we have that

$$C^s = \frac{1 - \chi^2}{1 - \chi} Y + \frac{\chi}{1 - \chi} \left( \frac{\Pi - 1 - i^b}{\Pi} b^b - \frac{\Pi - 1 + \gamma}{\Pi} m^b \right) \quad (74)$$

The savers money demand follows from

$$\Omega'(m^s) = \frac{i^s + \gamma}{1 + i^s} U''(C^s) \quad (75)$$

Finally, given the steady-state holdings of reserves and real money balances we can use the total money supply equation and the consolidated government budget constraint to solve for the remaining variables.
Log-linearized equilibrium conditions

We log linearize the non-linear equilibrium conditions around steady state, and define \( \hat{X} \equiv \frac{X_t - X}{X} \). For the intermediation cost function we assume the following functional form

\[
\Gamma \left( \frac{1}{l_t}, R_t, m_t, z_t \right) = \begin{cases} 
\left( \frac{l_t}{l_t^t} \right)^\nu (z_t)^{-\xi} + \frac{1}{2} (R_t - \bar{R})^2 + \frac{1}{2} (m_t - \bar{m})^2 & \text{if } R_t < \bar{R} \text{ and } m_t < \bar{m} \\
\left( \frac{l_t}{l_t^t} \right)^\nu (z_t)^{-\xi} & \text{if } R_t \geq \bar{R} \text{ and } m_t \geq \bar{m}
\end{cases}
\]

By combining the two Euler equations and the aggregate demand equation we derive the IS curve in equation (77), in which we define \( \sigma = \frac{1}{\bar{y}} \). By combining the five supply side equations we derive the Phillips curve in equation (86). We define the real interest rate \( \hat{r}^n_t \) in equation (78), and the interest rate spread in equation (87). We also use the market clearing condition to substitute \( \hat{b}_t \) for \( \hat{b}_t^b \). Hence, an equilibrium of the log-linearized model is a process for the 17 endogenous variables \( \{ \hat{c}_t, \hat{c}_s, \hat{b}_t^b, \hat{m}_t^b, \hat{m}_t^s, \hat{y}_t, \hat{r}^n_t, \hat{s}_t, \hat{\pi}_t, \hat{\sigma}, \hat{\omega}_t, \hat{\iota}_t^b, \hat{\iota}_t^s, \hat{\iota}_t^r \}^\infty \) such that the 17 equations listed below are satisfied. Note that the expressions for \( \hat{R}_t \) and \( \hat{m}_t \), in equations (90) and (91) respectively, only hold when the bank is not satiated.
\[ \dot{y}_t = E_t \dot{y}_{t+1} - \sigma \left( \dot{i}^s_t - E_t \pi_{t+1} - i^s_t \right) \]  
\[ \dot{r}^n_t = \dot{z}_t - E_t \dot{z}_{t+1} - \chi \dot{\omega}_t \]  
\[ \dot{c}^b_t = \dot{c}^b_{t+1} - \frac{1}{q_c^b} \left( \dot{i}^b_t - E_t \pi_{t+1} - \dot{\zeta} + E_t \zeta_{t+1} \right) \]  
\[ \dot{c}^s_t = \dot{c}^s_{t+1} - \frac{1}{q_c^s} \left( \dot{i}^s_t - E_t \pi_{t+1} - \dot{\zeta} + E_t \zeta_{t+1} \right) \]  
\[ c^b \pi_t + c^b \dot{c}^b_t = \pi_t (\chi y + b^b) + \chi y \dot{y}_t + b^b \dot{b}^b_t - \frac{b^b}{\pi} \dot{i}^b_{t-1} - (1 + i^b) \frac{b^b}{\pi} \dot{b}^b_{t-1} \]  
\[ \frac{\Omega''(m^b)m^b}{\Omega'(m^b)} \dot{m}^b_t = -q_c^b c^b_{t} - \frac{\dot{i}^b + \gamma - 1}{\dot{i}^b + \gamma} \dot{i}^b_t \]  
\[ \frac{\Omega''(m^s)m^s}{\Omega'(m^s)} \dot{m}^s_t = -q_c^s c^s_{t} - \frac{\dot{i}^s + \gamma - 1}{\dot{i}^s + \gamma} \dot{i}^s_t \]  
\[ c^{bc} c_t = \frac{R}{cbc} \dot{R}_t + \frac{m}{cbc} \dot{m}_t + \frac{m^b}{cbc} \dot{m}^b_t + \frac{m^s}{cbc} \dot{m}^s_t \]  
\[ \pi_t + \hat{c}^{bc} = \frac{1}{\pi} \dot{c}^{bc}_{t-1} + \frac{R}{cbc} \left( \dot{i}^s_{t-1} + i^s R_t \right) - \frac{\tau^s}{cbc} \left( \dot{\tau}^s + \dot{\pi}_t \right) \]  
\[ \hat{\pi}_t = \kappa \hat{y}_t + \beta E_t \hat{\pi}_{t+1} \]  
\[ \hat{i}^b_t = \hat{i}^s_t + \hat{\omega}_t \]  
\[ \hat{\omega}_t = \frac{\dot{i}^b - \dot{i}^s}{1 + i^b} \left( (\nu - 1) \dot{b}^b_t - \nu \dot{\hat{b}}_t - \nu \dot{\hat{z}}_t \right) \]  
\[ \hat{i}^s_t + \hat{\dot{z}}_t = \frac{\chi b^b}{(1 + i^s) z} \left( \dot{i}^b - \dot{i}^s + (i^b - i^s) \dot{b}^b_t \right) - \frac{R}{(1 + i^s) z} \left( \dot{\hat{i}}^s_t - \dot{\hat{i}}_t \right) \]  
\[ - \frac{m}{(1 + i^s) z} \left( \dot{i}^s_t + (i^s + \gamma) \dot{m}_t \right) - \frac{\Gamma}{(1 + i^s) z} \dot{i}^s_t \]  
\[ - \frac{\Gamma z}{\tau} \left( \nu (b^b_t - \hat{b}_t) + \nu \dot{\hat{z}}_t \right) - \frac{m (m - m)}{z} \dot{m}_t \]  
\[ \dot{R}_t = \frac{1}{(1 + i^s) R} \left( (\dot{R} - 1) \dot{i}^s_t + \dot{\hat{i}}^s_t \right) - \dot{\hat{i}}^s_t \]  
\[ \dot{m}_t = \frac{m - \hat{m}}{i^s - \gamma} \dot{\hat{i}}^s_t \]  
\[ \dot{\hat{i}}_t = \hat{r}^n_t + \phi_\Pi \hat{\pi}_t + \phi_Y \dot{y}_t \]  
\[ \dot{\hat{i}}^s_t = \max \left\{ -\gamma \beta^s - (1 - \beta^s), \hat{\dot{i}}^s_t \right\} \]
D Calibration and numerical simulation of a debt-deleveraging shock

Calibration

We pick the size of the preference shock to generate an approximately 4.5 percent drop in output on impact. This reduction in output is chosen to roughly mimic the average reduction in real GDP in Sweden, Denmark, Switzerland and the Euro Area in the aftermath of the financial crisis, as illustrated in Figure 15 in the Appendix. The drop in output in the US was of similar order. The persistence of the preference shock is set to generate a duration of the lower bound of approximately 12 quarters. We choose parameters from the existing literature whenever possible. We target a real borrowing rate of 4% and a real deposit rate of 1.5%, yielding a steady state credit spread of 2.5%. The preference parameter $q$ is set to 0.75, which generates an intertemporal elasticity of substitution of approximately 2.75, in line with Curdia and Woodford (2011). We set the proportional storage cost to 0.01, yielding an effective lower bound of -0.01%. This is consistent with the deposit rate being bounded at zero for most types of deposits, with the exception of slightly negative rates on corporate deposits in some countries. We set $\rho = 0.07$, which yields steady-state reserve holdings in line with average excess reserves relative to total assets for commercial banks from January 2010 and until April 2017. We set $\mu = 0.01$, implying that currency held by banks in steady state accounts for approximately 1.5 percent of total assets. This currency amount corresponds to the difference between total cash assets reported at US banks and total excess reserves from January 2010 until April 2017.

The parameter $\nu$ measures the sensitivity of the credit spread to private debt. We set $\nu$ so that a 1% increase in private debt increases the credit spread by 0.12%, as in Benigno, Eggertsson, and Romei (2014). Given the steady-state credit spread, $\tilde{I}$ pins down the steady-state level of private debt. We choose $\tilde{I}$ to target a steady state private debt-to-GDP ratio

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43 We pick the size of the debt deleveraging shock in the Appendix to generate a similar output reduction.

44 Detrended real GDP fell sharply from 2008 to 2009, before partially recovering in 2010 and 2011. The partial recovery was sufficiently strong to induce an interest rate increase. We focus on the second period of falling real GDP (which occurred after 2011), as negative interest rates were not implemented until 2014-2015. Targeting a reduction in real GDP of 4.5 percent is especially appropriate for the Euro Area and Sweden. Real GDP fell by somewhat less in Denmark, and considerably less in Switzerland. This is consistent with the central banks in the Euro Area and Sweden implementing negative rates because of weak economic activity, and the central banks in Denmark and Switzerland implementing negative rates to stabilize their exchange rates.

45 This is consistent with the average fixed-rate mortgage rate from 2010-2017. Series MORTGAGE30US in the St.Louis Fed’s FRED database.

46 We use series EXCSRESNS for excess reserves and TLAACBW027SBOG for total assets from commercial banks, both in the St.Louis Fed’s FRED database.
of approximately 95 percent, roughly in line with private debt in the period 2005 - 2015 (Benigno, Eggertsson, and Romei, 2014). The final parameter is $\iota$. In our baseline scenario we set $\iota = 0.2$. While $\iota$ is not important for our main result that negative interest rates are not expansionary, it is important for determining the feedback effect from bank profits to aggregate demand. In Table 3 in the next section we show how the potentially contractionary effect of negative interest rates depends quantitatively on $\iota$.

All parameter values are summarized in Table 4. Due to the occasionally binding constraint on $i^*_t$, we solve the model using OccBin (Guerrieri and Iacoviello, 2015) for the preference shock. For simplicity, we consider a cashless limit for the household’s problem 47.

**Debt deleveraging shock**

In this Appendix, we show the dynamic transition of our model to an alternative shock, a debt deleveraging shock. Specifically, we consider a permanent reduction in the debt limit $\bar{L}_t$, a shock often referred to as a “Minsky Moment” (Eggertsson and Krugman, 2012). The dynamic transition paths are shown in Figure 19. A permanent reduction in the debt limit directly increases the interest rate spread, causing the borrowing rate to increase. The initial increase in the borrowing rate is substantial, due to the shock’s impact on bank profits and the feedback effect via $\iota$. In the frictionless case, the central bank can perfectly counteract this by reducing the reserve rate below zero. Given the bound on the deposit rate however, the central bank looses its ability to bring the economy out of a recession. Any attempt at doing so, by reducing the reserve rate below zero, only lowers bank profits and aggregate demand further.

In some respects this shock – with the associated rise in the borrowing rate – resembles more the onset of the financial crisis, when borrowing rates (in some countries) increased. The preference shock considered in the main body of the text is more consistent with the situation further into the crisis, when both deposit and lending rates were at historical low levels (perhaps reflecting slower moving factors such as those associated with secular stagnation, see Eggertsson and Mehrotra (2014)). From the point of view of this paper however, it makes no difference which shock is considered in terms of the prediction it has for the effect of negative central bank rates. In both cases, the policy is neutral when there is no feedback from bank profits, and contractionary when there is such a feedback.

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47 There are no additional insights provided by allowing households to hold money in the numerical experiments, even if this feature of the model was essential in deriving the bound on deposits.

48 A temporary shock to banks’ intermediation costs yield qualitatively similar results.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverse of Frisch elasticity of labor supply</td>
<td>$\eta = 1$</td>
<td>Justiniano et.al (2015)</td>
</tr>
<tr>
<td>Preference parameter</td>
<td>$q = 0.75$</td>
<td>Yields IES of 2.75(Curdia and Woodford, 2011)</td>
</tr>
<tr>
<td>Share of borrowers</td>
<td>$\chi = 0.61$</td>
<td>Justiniano et.al (2015)</td>
</tr>
<tr>
<td>Steady-state gross inflation rate</td>
<td>$\Pi = 1.005$</td>
<td>Match annual inflation target of 2%</td>
</tr>
<tr>
<td>Discount factor, saver</td>
<td>$\beta^s = 0.9901$</td>
<td>Annual savings rate of 1.5%</td>
</tr>
<tr>
<td>Discount factor, borrower</td>
<td>$\beta^b = 0.9963$</td>
<td>Annual real borrowing rate of 4 %</td>
</tr>
<tr>
<td>Probability of resetting price</td>
<td>$\alpha = 2/3$</td>
<td>Gali (2008)</td>
</tr>
<tr>
<td>Taylor coefficient on inflation gap</td>
<td>$\phi_\Pi = 1.5$</td>
<td>Gali (2008)</td>
</tr>
<tr>
<td>Taylor coefficient on output gap</td>
<td>$\phi_Y = 0.5/4$</td>
<td>Gali (2008)</td>
</tr>
<tr>
<td>Elasticity of substitution among varieties of goods</td>
<td>$\theta = 7.88$</td>
<td>Rotemberg and Woodford (1997)</td>
</tr>
<tr>
<td>Proportional storage cost of cash</td>
<td>$\gamma = 0.01$</td>
<td>Effective lower bound $i_s^* = -0.01%$</td>
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<td>Reserve satiation point</td>
<td>$R = 0.07$</td>
<td>Target steady-state reserves/total assets of 13 %</td>
</tr>
<tr>
<td>Money satiation points</td>
<td>$\bar{m} = 0.01$</td>
<td>Target steady-state cash/total assets of 1.5 %</td>
</tr>
<tr>
<td>Marginal intermediation cost parameters</td>
<td>$\nu = 6$</td>
<td>Benigno, Eggertsson, and Romei (2014)</td>
</tr>
<tr>
<td>Level of safe debt</td>
<td>$\bar{I} = 1.3$</td>
<td>Target debt/GDP ratio of 95 %</td>
</tr>
<tr>
<td>Link between profits and intermediation costs</td>
<td>$\iota = 0.2$</td>
<td>1 % increase in profits $\approx 0.01 %$ reduction in credit spread</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Shock</th>
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<tbody>
<tr>
<td>Preference shock</td>
<td>2.5 % temporary decrease in $\zeta_t$</td>
<td>Generate a 4.5 % drop in output on impact</td>
</tr>
<tr>
<td>Persistence of preference shock</td>
<td>$\rho = 0.9$</td>
<td>Duration of lower bound of 12 quarters</td>
</tr>
<tr>
<td>Debt deleveraging shock (in Appendix D)</td>
<td>50 % permanent reduction in $\bar{I}$</td>
<td>Generate a 4.5 % drop in output on impact</td>
</tr>
</tbody>
</table>

Table 4: Parameter values
Figure 19: Impulse response functions following a debt deleveraging shock (a permanent reduction in $\zeta_t$), under three different models. **Standard model** refers to the case where there is an effective lower bound on both deposit rates and the central bank’s policy rate. **No bound** refers to the case where there is no effective lower bound on any interest-rate. **Negative rates** refers to the model outlined above, where there is an effective lower bound on the deposit rate but no lower bound on the policy rate.